MMS-15X

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From CY 2020 to AY 2020 - CY 2021]

First Year

ALGEBRA

Time: 3 Hours **Maximum Marks: 70**

PART - A (5 x 5 = 25 Marks)

Answer any FIVE questions out of Eight Questions in 300 words

1. If H is a nonempty finite subset of a group G and H is closed under multiplication, then show that H is a subgroup of G.
2. State and prove the division algorithm for polynomial rings.
3. If $u, v \in V$, then show that $|u, v| \leq \|u\|\|v\|$.
4. Let G be a group of automorphisms of a field K. Show that the fixed field of G is a subgroup of K.
5. If $T \in \mathrm{A}(V)$ is nilpotent, then $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$ where $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.
6. If G is a finite group and $a \in G$, then show that $\mathrm{O}(a)|\mathrm{O}(G)$.
7. State and prove the Gauss lemma.
8. If H and K are finite subgroup of G of orders $o(H)$ and $o(K)$, respectively, then show that
$$o(HK) = \frac{o(H)\ o(K)}{o(H \cap K)}.$$

PART - B **(3 x 15 = 45 Marks)**

Answer any THREE questions out of Five Questions in 1000 words

9. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field.
10. If V is finite-dimensional vector space and if W is a subspace of V, then show that W is finite dimensional and $\dim V/W = \dim V - \dim W$.
11. If F is of characteristic 0 and if a, b are algebraic over F, then show that there exists an element $c \in (a, b)$ such that $F(c) = F(a, b)$.
12. If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then T satisfies a polynomial of degree n over F.
13. If p is a prime number and $p|(G)$, then show that G has an element of order p.

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From AY 2003 to AY 2019]

First Year

ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

PART - A **(3 x 5 = 15 Marks)**

Answer any THREE questions out of Five Questions in 300 words.

1. State and prove Lagrange's theorem.

2. Prove that HK is a subgroup of G iff HK = KH.

3. Prove that a subgroup H of a group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H$, $g \in G$.

4. Prove that a non zero finite integral domain is a field.

5. In any vector space V (F), the following results hold:
   (i) $o.x = 0$
   (ii) $\alpha.0 = 0$
   (iii) $(-\alpha)x = -(\alpha x) = \alpha(-x)$
   (iv) $(\alpha-\beta)x = \alpha x - \beta x$, $\alpha, \beta \in F$, $x \in V$

PART - B **(4 x 15 = 60 Marks)**

Answer any FOUR questions out of Seven Questions in 1000 words.

6. A) Prove that a subgroup H of a group G is normal subgroup of G if product of two right cossets of H in G is again a right cosset of H in G.

   B) State and prove Sylow's First Theorem.

7. State and prove Fundamental theorem of group homomorphism.

8. A) Prove that a commutative ring R is an integral domain iff for all $a,b,c \in R$ $(a \neq 0)$ $ab = ac \Rightarrow b=c$.

   B) Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.

9. Prove that if R is an integral domain with unity in which every non-zero, non unit element is a finite product of irreducible elements and every irreducible element is prime, then R is a UFD.

10. Let R be a commutative ring with unity. Then prove that an ideal M of R is maximal ideal of R iff R/M is a field.

11. A) If V is finite-dimensional and if W is a subspace of V, then W is finite-dimensional, dim W $\leq$ dim V and dim V / W = dim V –dim W.

B) Let V and W be two vetor spaces (over F) of dim m and n respectively. Then prove that Hom (V, W) has dim mn.

12. A) If V is finite-dimensional over F then prove that T$\in$ A (V) if and only if the constant term of the minimal polynomial for T is not 0.

B) If V is n-dimenstional over F and if T $\in$ A (V) has all its characterstic roots in F then prove that T satisfies a polynomial of degree " n" over F.

SUBJECT CODE: MMS-16X

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From CY 2020 to AY 2020 - CY 2021]

First Year

REAL ANALYSIS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions out of Eight Questions in 300 words

1. Show that any compact subset of a metric space is closed.
2. If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connectedsubset of X, then show that $f(X)$ is connected.
3. Let f be areal differentiable function on $[a, b]$ and let $f'(a) < \lambda < f'(b)$. Show that there is a point $x \in (a, b)$ such that $f'(x) = \gamma$.
4. If $x > 0$ and $y > 0$, then show that

$$\int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

5. Show that a linear operator A on a finite-dimensional vector space X is one-to-one if and onlyif the range of A is all of X.
6. Let $\{s_n\}, \{t_n\}$ be complex sequences and let $\lim_{n\to\infty} s_n = s$ and $\lim_{n\to\infty} t_n = t$ Show that

$$\lim_{n\to\infty} (s_n + t_n) = s + t.$$

7. Show that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P suchthat

$$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon.$$

8. Show that a subset E of real line R^1 is connected if and only if it has the following property:If $x \in E$, $y \in E$ and $x < z < y$, then $z \in E$.

PART - B **(3 x 15 = 45 Marks)**

Answer any THREE questions out of Five Questions in 1000 words

9. A mapping f of a metric space X into a metric space Y is continuous if and only if $f^{-1}(V)$ isopen in X for every open set V in Y.
10. State and prove the Stone-Weierstrass theorem.
11. If X is a complete metric space, and if ϕ is a contraction of X, then show that there exists one and only one $x \in X$ such that $\phi(x) = x$).
12. State and prove the root test.
13. Let f be a continuous mapping of a compact metric space X into a metric space Y. Show that f is uniformly continuous.

SUBJECT CODE: MMS-16 / PGDMAT-12

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From AY 2003 to AY 2019]

First Year

REAL ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(3 x 5 = 15 Marks)**

Answer any THREE questions out of Five Questions in 300 words.

1. In the Euclidian space prove that the following are true
   a) $|x| \geq 0$
   b) $|x| = 0$ iff $x = 0$
   c) $|\alpha x| = |\alpha| |x|$
   d) $|xy| \leq |x| |y|$
   e) $|x+y| \leq |x|+|y|$
   f) $|x-z| \leq |x - y|+|y - z|$

2. Prove that countable union of countable set is countable

3. Prove that composition of two continuous function is continuous

4. State and prove the fundamental theorem of calculus

5. If $f_1 \in R(\alpha)$ ON $[a,b]$ and $f_2 \in R(\alpha)$ on $[a,b]$ and $f_1 \geq f_2$ on $[a, b]$ then prove that $\int_a^b f_1(x)dx \leq \int_a^b f_2(x)dx$.

PART - B **(4 x 15 = 60 Marks)**

Answer any FOUR questions out of Seven Questions in 1000 words.

6. State and prove Merten's theorem

7. Suppose $\{S_n\},\{p_n\},\{t_n\}$, are complex sequences and $\lim_{n\to\infty} s_n = s$ and $\lim_{n\to\infty} t_n = t$.
   Prove the following

$\lim_{n\to\infty}$ $s_n + t_n = s + t$

$\lim_{n\to\infty}$ $cs_n = cs$

$\lim_{n\to\infty}$ $s_n\, t_n = st$

$\lim_{n\to\infty} \frac{1}{sn} = \frac{1}{s}$ Provided $S_n \neq 0$ and $S \neq 0$

8. A) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that " f " is uniformly continuous

   B) Prove that continuous image of connected set is connected

9. Suppose f and g are defined on $[a,b]$ and are differentiable at a point $x \in [a, b]$.
   Then prove that a) $(f+g)$ '(X) = f '(X)+ g '(X)
   b) (fg) '(x) = f '(X)g (X) + f (X) g '(X)
   c) $(f/g)^1\ (x) = \frac{g(x)f^1(x)-f(x)g^1(x)}{g^2(x)}$

10. (A) State and prove weiestrass M – test for uniform convergence of series of functions

    B) Prove that A sequence of functions $\{ f_n \}$ converges to f with respect to C(X) $IF\ F_n \to f\ uniforms\ on\ x.$

11. A) Show that differentiable functions are continuous

    B) State and prove the root test

12. With usual notation prove the following

    A) E (z +x) = E ((z) (w)) where z and w are complex

    B) E (Z₀. E (-Z)= 1

    C) E '(z) = e (z)

    D) E (nz) = e (z)n

SUBJECT CODE: MMS-17

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From AY 2003 to AY 2019]

First Year

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART - A (3 x 5 = 15 Marks)

Answer any THREE questions out of Five Questions in 300 words.

1. Find the singularities of the function $\frac{\cot \pi z}{(z-a)^3}$.

2. Using residue theorem, evaluate $\int_C \frac{z^2+1}{(z-1)(z-2)} dz$ where C is $|z| = 3$.

3. Define conformal mapping. If $\omega = x + \frac{iby}{a}, 0 < a < b,$ prove that the inside of the circle $x^2 + y^2 = a^2$ corresponds to the inside of an ellipse in the $\omega - plane.$ Is the transformation conformal?

4. Using Lagrange's formula of interpolation find $y(0.9)$ given

x	7	8	9	10
y	3	1	1	9

5. Solve $\frac{dy}{dx} = x + y$ given $y(0) = 1.$ Obtain the value of $y(0.1)$ using Picard's method.

PART - B (4 x 15 = 60 Marks)

Answer any FOUR questions out of Seven Questions in 1000 words.

6. State and prove Rouchy's theorem.

7. Discuss the transformation $\omega = \sin z$.

8. Find an iterative formula to find the reciprocal of a number N using Newton's method and hence find the value of $\frac{1}{19}$.

9. Apply Gauss Jordan method to find the solution of the following system:

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

10. Evaluate the integral $\int_{4}^{5.2} \log_e x \, dx$ using Trapezoidal rule and Simpson's rules.

11. Find the value of $\cos(1.74)$ from the following table:

x	1.7	1.74	1.78	1.82	1.86
sin x	0.9916	0.9857	0.9781	0.9691	0.9584

12. Determine the value of $y(0.4)$ using Milne's method given $\frac{dy}{dx} = xy + y^2, y(0) = 1$

Use Taylor series to get the values of $y(0.1), y(0.2)$ and $y(0.3)$.

SUBJECT CODE: MMS-17X

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From CY 2020 to AY 2020 - CY 2021]

First Year

COMPLEX ANALYSIS AND NUMERICAL METHODS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions out of Eight Questions in 300 words

1. If $f(z)$ is a regular function z. Prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\ |f(z)|^2 = 4\ |f'(z)|^2.$$

2. Evaluate $\int_C \frac{z}{(9-z^2)(z+i)}\,dz$, where C is the circle $|z| = 2$ de scribes in positive sense.

3. Find a positive root of $xe^x = 2$ by the method of False position.

4. Using the following data, determine by Lagrange's interpolation formula the percentage of criminals under 35 years.

Age (under years) :	25	30	40	50
% Number of criminals :	52.0	67.3	84.1	94.4

5. Using Taylor's series method, find the value of $y(0.2)$ given that $\frac{dy}{dx} = 1 - 2xy$, and $y(0) = 0$.

6. Find the bilinear transformation which maps $z = 1, i, -1$ onto $w = i, 0, -i$ respectively.

7. Expand $f(z) = \frac{z+3}{z\,(z^2-z-2)}$ in powers of z, where (i) $|z| < 1$, (ii) $1 < |z| < 2$.

8. State and prove the necessary condition for $f(z)$ to be analytic.

PART - B **(3 x 15 = 45 Marks)**

Answer any THREE questions out of Five Questions in 1000 words

9. State and prove the Taylor's theorem.

10. Solve the following system of equations by Gauss Seidel method.

$$28x + 4y - z = 32, \quad x + 3y + 10z = 24, \quad 2x + 17y + 4z = 35$$

11. The population of a town is as follows.

Year :	1941	1951	1961	1971	1981	1991
Population in lakhs :	20	24	29	36	46	51

Using Newton's interpolation, estimate the population increase during the period 1946 to 1976.

12. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2-x^2}$ given $y(0) = 1$ at $x = 0.2$.

13. Discuss the transformation w=sin z.

Subject Code:	**MMS-18**

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From AY 2003 to AY 2019]

First Year

MATHEMATICAL STATISTICS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(3 x 5 = 15 Marks)**

Answer any THREE questions out of Five Questions in 300 words.

1. Define Binomial distribution and find its mean and variance.
2. State weak Law of large numbers.
3. Show that the sample mean $\bar{x}$ is an unbiased estimator of $\frac{1}{\theta}$ for the distribution

   $f(x,\theta) = \theta(1-\theta)^{x-1} \quad x = 1,2,....,0 < \theta < 1.$
4. A coin is tossed 6 times . The Null hypothesis H_0 :p=0.5 is rejected if 5 or more trails result in heads. Obtain the power of this test if H_0 :p=0.5 is tested against H_1:p$\neq$0.5 where p denotes probability of getting a head in any trial.
5. Independent random samples of sizes 20 and 25 taken from $N(\mu_1 : \sigma_1 = 3)$ and $N(\mu_2 : \sigma_2 = 4)$ have means 50 and 45 respectively. Construct a 90% confidence Interval for $2(\mu_1 - \mu_2)$.

PART - B **(4 x 15 = 60 Marks)**

Answer any FOUR questions out of Seven Questions in 1000 words.

6. State and Prove Tchebychev's inequality.
7. Find the Moment generating function of Normal distribution and hence find its mean and variance.
8. Two stochastic variables X and Y take the values 1,2,3 and their probabilities are as follows

Y X	1	2	3
1	0.1	0.1	0.1
2	0.1	0.2	0.1
3	0.1	0.1	0.1

Find E(X),E(Y),Var(X),Var(Y),E(XY),E(X+Y)and Cov(X,Y).

9. Show that the sample variance is a consistent estimator for the population Variance of a normal distribution.
10. State and Prove Central limit theorem
11. State and Neyman Pearson Lemma.
12. State and prove Rao Cramer inequality.

Subject Code:	**MMS-18X**

P.G. DEGREE EXAMINATION – JANUARY 2022

MATHEMATICS

[From CY 2020 to AY 2020 - CY 2021]

First Year

MATHEMATICAL STATISTICS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions out of Eight Questions in 300 words

1. Given the pdf of a continuous random variable X as follows
$$f(x) = \begin{cases} kx(1-x & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$$
Find k and the cumulative distribution function.

2. State the properties of t-distribution.

3. Find the MGF of the Poisson distribution and hence obtain its mean and variance.

4. Given that $f(x,\theta) = (1+\theta)x^{\theta}, \theta > 0, 0 \leq x < 1$. If the hypothesis $H_0\!: \theta = 2$ is to be tested by a single observation on X using the critical region $X \leq 0.25$. (i) Find α, (ii) If $H_1\!: \theta = 3$ find β.

5. In a sample of 500 people in Kerala are tea drinkers and the rest are coffee drinkers. Can we assume that both coffee and tea are equally popular in the state at 5% level of significance?

6. Show that limiting form of t-distribution is normal distribution.

7. Briefly explain about the characteristics of estimators.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with p.d.f.
$$f(x,\theta) = \theta x^{\theta-1}; 0 < x < 1, \theta > 0.$$
Show that $t_1 = \prod_{i=1}^{n} X_i$, is sufficient for θ.

PART - B **(3 x 15 = 45 Marks)**

Answer any THREE questions out of Five Questions in 1000 words

9. State and prove Chebychev's in equlity.

10. Find the moment generating function of Gamma distribution and also find its mean and variance.

11. Find the probability density function of F distribution.

12. State and prove Rao-Blackwell theorem.

13. Test the hypothesis $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 < \theta_0$ based on a sample of size n from the population with density function.

$$f(x, \theta) = (1 + \theta)x^{\theta}, 0 < x < 1.$$