P.G. DEGREE EXAMINATION - JUNE 2021

FIRST YEAR

MATHEMATICS

ALGEBRA

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions in 50 words
[All questions carry equal marks]

1. Define normalizer of an element of a group.

2. If R is a ring, then for any $a \in R$ show that $a0 = 0\,a = 0$.

3. Define an R-module.

4. Define a normal extension of a field.

5. When do we say that two linear transformations are similar?

PART - B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions out of Seven questions in 150 words
[All questions carry equal marks]

6. Show that N is a normal subgroup of a group of a group G if and only if $gNg^{-1} = N$ for every $g \in G$.

7. State and prove the unique factorization theorem.

8. If V is a finite-dimensional inner product space and W is a subspace of V, then show that $(W^{\perp})^{\perp} = W$.

9. State and prove the remainder theorem.

10. If $T \in A(V)$ is nilpotent, then $\alpha_0 + \alpha_1 T + \cdots + \alpha_m T^m$ where $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.

11. If $o(G) = p^2$ where p is a prime, then show that G is abelian.

12. Let S be a non-empty subset of a vector space V. Show that the linear span of S is a subgroup of V.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions out of Seven questions in 400 words
[All questions carry equal marks]

13. State and prove Cayley's theorem.

14. Show that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R.

15. State and prove the Schwarz inequality.

16. Show that the number e is transcendental.

17. If $T \in A(V)$ has all its characteristic roots in F, then there is a basis of V in which the matrix of T is triangular.

18. Show that the number of p-sylow subgroups in G, for a given prime, is of the form $1 + kp$.

19. Let R be a commutative ring with unit element whose only ideas are (0) and R itself. Show that R is a field.

P.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 3 = 15 Marks)**

Answer any **FIVE** of the following.

1. Let G be a group. Then prove that (a) identity element is unique, (b) every Element $a \in G$ has a unique inverse in G.

2. Show that a non empty subset H of a group G is a subgroup of G if and only if a, b∈H $ab^{-1} \in H$.

3. Prove that every group G is isomorphic to a permutation group.

4. Prove that a non zero finite integral domain is a field.

5. If $\theta : R \to R^{'}$ be a homomorphism, then prove that (i) $\theta(0) = 0^{'}$ (ii) $\theta(-a) = -\theta(a)$ where $0, 0^{'}$ are zeros of the rings R and R respectively.

6. If u,v∈V then prove that |(u,v)|=$\|u\|\|v\|$.

7. If p(x) ∈ F[x] is solvable by radicals over F, then prove that the Galois group over F of p(x) is a solvable group.

8. If T∈A(V) has all its characteristics roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

PART - B **(5 x 12 = 60 Marks)**

Answer any **FIVE** of the following.

9. Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Then prove that $G/K \approx \bar{G}$.

10. A) Prove that "Two abelian groups of order pn are isomorphic if and only if they have the same invariants"

    B) Prove that in a ring R, the following results hold

    (i) $a \cdot 0 = 0 \cdot a = 0$ for all a$\in R$

    (ii) $a(-b) = (-a)b = -ab$ for all a,b∈R

    (iii) $(-a)(-b) = ab \; \forall \; a, b \in R$

    (iv) $a(b - c) = ab - ac \; \forall \; a, b, c \in R$

11. Let R be a commutative ring with unity, then prove that an ideal M of R is maximal ideal of R iff R/M is a field.

12. Prove that if R is an integral domain with unity in which every non-zero, non unit element is a finite product of irreducible elements and every irreducible element is prime, then R is a UFD.

13. Prove that two finite dimensional vector space over F are isomorphic, if and only if they have the same dimension.

14. If V is finite-dimensional and if W is a subspace of V, then W is finite-dimensional, $\dim W \leq \dim V$ and $\dim V/W = \dim V - \dim W$.

15. A) If V is finite dimensional over F then prove that $T \in A(V)$ if and only if the constant term of the minimal polynomial for T is not 0
B) If V is n-dementional over F and if $T \in A(V)$ has all its characteristics roots in F then prove that T satisfies a polynomial of degree "n" over F

16. If V is finite dimensional over F then for S, $T \in A(V)$
$r(ST) \leq r(T)$
$r(TS) \leq r(T)$ so that $r(ST) \leq \min \{r(T), r(S)\}$
$r(ST) = r(TS) = r(T)$ for S regular in A(V)

P.G. DEGREE EXAMINATION - JUNE 2021

FIRST YEAR

MATHEMATICS

REAL ANALYSIS

Time : 3 Hours **Maximum Marks : 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions in 50 words
[All questions carry equal marks]

1. Show that every neighborhood is an open set.

2. When do you say that a function *f* has a simple discontinuity?

3. State the chain rule for differentiation.

4. Define uniform convergence of sequence of functions.

5. State the contraction principal.

PART - B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions out of Seven questions in 150 words
[All questions carry equal marks]

6. Show that a set E is open if and only if its complement is closed.

7. Let *f* be a continuous mapping of a compact metric space X into a metric space Y . Show that $f(X)$ is compact.

8. State and prove the fundamental theorem of calculus.

9. If $x > 0$ and $y > 0$, then show that
$$\int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

10. Let $\boldsymbol{f}$ maps a convex open set $E \subset R^n$ into R^m, and let $\boldsymbol{f}$ is differentiable in E and there is a real number M such that $\|f'(x)\| \le M$ for every $x \in E$. Show that
$$|\boldsymbol{f}(\boldsymbol{b}) - \boldsymbol{f}(\boldsymbol{a})| \le \boldsymbol{M}|\boldsymbol{b} - \boldsymbol{a}|$$
for every $\boldsymbol{a} \in E, \boldsymbol{b} \in E$.

11. In any metric space X, show that every convergent sequence is a Cauchy sequence.

12. Let *f* be a function on $[a, b]$. If *f* has a local maximum at a point $x \in (a, b)$ and if

$f^{'}(x)$ exists, then show that $f^{'}(x) = 0$.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions out of Seven questions in 400 words
[All questions carry equal marks]

13. Show that every k-cell is compact.

14. Let f be a continuous mapping of a compact metric space X into a metric space Y. Show that f is uniformly continuous.

15. Assume that α increasing monotonically and $\alpha^{'} \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Show that $f \in R[\alpha]$ if and only if $f\alpha^{'} \in R$. Also show that

$$\int_a^b f d\alpha = \int_a^b f(x)\alpha^{'}(x)dx$$

16. State and prove the Cauchy criterion for uniform convergence.

17. State and prove the inverse function theorem.

18. Show that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e$.

19. If $\gamma^{'}$ is a continuous on $[a, b]$, then show that γ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma^{'}(t)|dt$$

P.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

REAL ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART – A **(5 x 3 = 15 Marks)**

Answer any **FIVE** questions.

1. Prove that Q is an ordered field which do not have both least upper bound and greatest lower bound property.

2. Show that a set E is open if and only if E^c is closed.

3. Prove that open balls, closed balls and k-cells are convex sets in R^k.

4. Prove that continuous image of compact space is compact.

5. State and prove the fundamental theorem of calculus.

6. If $\{f_n\}$ is a point wise bounded sequence of complex functions on a countable set E, then prove that $\{f_n\}$ has a subsequence $\{f_{nk}\}$ such that $\{f_{nk}(x)\}$ convergence for every $x \in E$.

7. Prove that $\lim_{n\to\infty}(1+\frac{x}{n})^n = e^x$

8. Prove that $L(R^n, R^m)$ is a metric space

PART - B **(5 x 12 = 60 Marks)**

Answer any **FIVE** questions.

9. Prove that every k-cell in R^c is compact

10. State and prove Heine Boral theorem

11. A) Let X and Y be metric space. Suppose $E \subset X$, f maps E into Y and p is a limit point of E then prove that $\lim_{x\to p} f(x) = q$ if and only if $\lim_{n\to\infty} f(p_n) = q$. For every sequence $\{p_n\}$ in E such that $p_n \neq p$ and if $\lim_{n\to\infty}(p_n) = q$.

    B) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y

12. A) Prove that continuous image of connected set is connected.
B) Prove that composition of two continuous function is continuous

13. Let f be a bounded real function defined by [a,b]. Then prove that $f \in R(\alpha)$ if and only if for every $\varepsilon > 0$ there exists a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha < \varepsilon)$$

14. a) If $f_1 \epsilon R(\alpha)$ on [a,b] and $f_2 \epsilon R(\alpha)$ on [a,b] and $f_1 \leq f_2$ on [a,b] then prove that $\int_a^b f_1(x)dx \leq \int_a^b f_2(x)dx$
b) State and prove Cauchy criterion for uniform convergence.

15. With usual notation prove the following
A) $E(z + x) = E((z)(w))$ where z and w are complex
B) $E(z) \cdot E(-z) = 1$
C) $E^{'}(z) = e(z)$
D) $E(nz) = e(z)^n$

16. State and prove implicit function theorem.

P.G. DEGREE EXAMINATION - JUNE 2021
MATHEMATICS
FIRST YEAR
COMPLEX ANALYSIS AND NUMERICAL METHODS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions.

1. Define a bilinear transformation.

2. Find the residue of $\frac{z^2}{z^2+a^2}$ at $z = ia$.

3. What is the condition for the convergence of the iterative method for solving $x = \emptyset(x)$?

4. Write down Lagrange's interpolation formula.

5. Using Euler's method find $y(0.1)$ given $y' = -y, y(0) = 1$.

PART – B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions.

6. Prove that the function $u = y^3 - 3x^2y$ is a harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ in terms of z.

7. State and prove the Liouville's theorem.

8. Using Newton-Raphson method, find a root of $4x - e^x = 0$ that lies between 2 and 3.

9. Given the following data, find $y'(6)$ and maximum value of y.

x	0	2	3	4	7	9
y	4	26	58	112	466	922

10. Solve $\frac{dy}{dx} = x + y$, given $y(0) = 1$ by picard's methods and obtain the value of $y(0.1)$.

11. Find a bilinear transformation which maps the points $z = \infty, i, 0$ into points $w = 0, i, \infty$ respectively.

12. Evaluate $\int_c \frac{e^{2z}dz}{(z+1)^4}$ where C is $|z| = 3$.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions.

13. Discuss the transformation $w = z^2$.

14. State and prove Cauchy's integral formula.

15. Solve the following system of equations by using Gauss-Seidel method.
$8x - 3y + 2z = 20, 4x + 11y - z = 33$ and $6x + 3y + 12z = 35$.

16. Evaluate $\int_4^{5.2} log_e x dx$ using (i) Trapezoidal Rule, (ii) Simpson's one-third rule, (iii) Simpson's three-eight rule.

17. Using Runge-Kutta method of fourth order, find $y(0.7)$ correct to four decimal places if $y' = y - x^2, y(0.6) = 1.7379$.

18. Expand in the series the function $f(z) = \frac{1}{z^2-3z+2}$ in the regions.
(i) $|z| < 1$, (ii) $1 < |z| < 2$, (iii) $|z| > 2$.

19. Using Newton's interpolation formula, find the values of y at $x = 21$ and $x = 28$ from the following data:

x	:	20	23	26	29
y	:	0.3420	0.3907	0.4384	0.4848

PG-A-791	**MMS-25**

P.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE of the following.

1. Define a topology and the topology generated by a basis.

2. Show that every finite point set in a Hausdorff space X is closed.

3. Show that every closed subspace of a compact space is compact.

4. State and prove intermediate value theorem.

5. Prove that a subspace of a completely regular is regular.

6. State and prove Minkowski's inequality.

7. Let S be a non-empty subset of a Hilbert space H, show that (i) $S \subset S^{\perp\perp}$ and (ii) $S \cup S^{\perp} = \{0\}$

8. For any operator T, show that T^*T and TT^* are positive.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Let X be a topological space. Then the following conditions hold.
   (i) $X, \emptyset$ are closed.
   (ii) Arbitrary intersection of closed sets is closed.
   (iii) Finite union of closed sets is closed.

10. Show that topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

11. Show that product of finitely many compact spaces is compact.

12. Prove that every metrizable space is normal.

13. If N is a normed linear space and x_0 is a non-zero vector in N, then show that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$

14. State ad prove closed graph theorem

15. State and prove Bessel's inequality in general form.

16. If T is an operator on H for which $(Tx, x) = 0$ for all x, then show that $T = 0$.

P.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

OPERATIONS RESEARCH

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Write the dual of the LPP $Min\ Z = 4x_1 + 6x_2 + 18x_3$ subject to $x_1 + 3x_2 \geq 3$, $x_2 + 2x_3 \geq 5$ and $x_1, x_2, x_3 \geq 0$.

2. Write the differences between CPM and PERT.

3. Construct the network for the project whose activities and their relationship are as given below.
Activities: $A < C, D; B < C, D; C < E; D, E < F$.

4. Solve the following 2 x 2 game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}. \end{array}$$

5. What are the main characteristics of game theory.

6. What are the customer's behaviour in Queuing system.

7. Solve the NLPP $Max\ Z = 4x_1 - x_1^2 + 8x_2 - x_2^2$ subject to $x_1 + x_2 = 2$, $x_1, x_2 \geq 0$.

8. Find the stationary points of $f(x) = 4x^4 - x^2 + 5$ and determine the nature of the stationary points.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Solve using Simplex method.
$Min\ Z = 8x_1 - 2x_2$ Subject to
$-4x_1 + 2x_2 \leq 1$, $5x_1 - 4x_2 \leq 3$ and $x_1, x_2 \geq 0$.

10. Draw the network for the following PERT network and find the critical path. What is the probability that the project will be completed in 27 days?

Activity:	1-2	1-3	1-4	2-5	2-6	3-6	4-7	5-7	6-7
Optimistic Time (days):	3	2	6	2	5	3	3	1	2
Pessimistic Time (days):	15	14	30	8	17	15	27	7	8
Most likely time (days):	6	5	12	5	11	6	9	4	5

11. Solve the following game by using simplex method

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix} \end{array}$$

12. Find the optimum solution to the following L.P.P $Max\ Z = x_1 + x_2$ subjet to constraints $3x_1 + 2x_2 \leq 5, x_2 \leq 2$ and $x_1, x_2 \geq 0$ and are integers.

13. State and prove Pollaczek-Khinthcine formula.

14. Cars arrive at a petrol pump, have one petrol unit, in Poission fashion with an average of 10 cars per hour. The service is distributed exponentially with a mean of 3 minutes. Find
    (i) average number of cars in the system.
    (ii) average waiting time in the Queue.
    (iii) average Queue length.
    (iv) the probability that the number of cars in the system is 2.

15. $Min\ Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$
subject to $x_1 + x_2 + x_3 = 25$, $2x_1 - x_2 + 2x_3 = 15$ $x_1, x_2, x_3 \geq 0$

16. Use dynamic programming solve $Max\ Z = y_1 \cdot y_2 \cdot y_3$
subject to the constraints $y_1 + y_2 + y_3 = 5$ and $y_1, y_2, y_3 \geq 0$.

PG-A-793	MMS-27

P.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

GRAPH THREORY AND ALGORITHMS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions.

1. Prove that an undirected graph is a tree, if and only if, there is unique simple path between every pair of vertices.
2. Define distance, diameter and radius of a graph.
3. Explain connectivity and edge connectivity.
4. Explain 1-Isomorphic graphs and 2-Isomorphic graphs.
5. Show that every 3-regular graph without cut edges has a perfect matching.
6. Write down Fleury's algorithm.
7. Explain Hajos conjecture.
8. Mention some of the properties of planar and non-planar graph.

PART - B **(5 x 10 = 50 Marks)**

Answer any FIVE questions.

9. State and prove Turan's theorem.
10. Prove that a graph is bipartite if and only if it contains no odd cycle.
11. State and prove Menger's theorem.
12. Prove that a non empty connected graph is Eulerian if and only if it has no vertices of odd degree.
13. Prove that if G is a k-regular bipartite graph with $k > 0$, then G has a perfect matching.
14. State and prove vizing's theorem.
15. State and prove DMP planarity algorithm.
16. Prove that every planar graph is five colourable.