PG-C-787 PGDMAT-11

M.SC DEGREE EXAMINATION - DECEMBER 2020

Mathematics

First Year

ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

Section A (5 x 5 = 25 Marks)

Answer any **FIVE** questions:

1. Suppose H and K are two subgroup of G, then prove that if HK is subgroup of G if and only if HK=KH.

2. Prove that subgroup of every cyclic group is cyclic.

3. Prove that the subgroup N of G is normal subgroup of G if and only if every right coset of N in G is left coset N in G.

4. Define field and integral domain and prove that every filed is an integral domain.

5. If U,V are ideals of R, then prove that U + V are also ideals of R.

6. Prove that if R is a Eculidean ring, then any two element a, b in R have a greatest common divisor d, then $d = ta + sb$, $s,t \in$ R.

7. If v_1, v_2,…v_n $\in$ V are linearly independent vectors, then every element in the linear spam have a unique representation of the form $\lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n$, where each $\lambda_i \in$ F.

8. If V and W are vector spaces with dimension m and n respectively over F, then prove that Hom (V, W) is dimension mn over F.

Section B (5 x 10 = 50 Marks)

Answer any **FIVE** questions.

9. State and prove the first part of sylows theorem.

10. State and prove Cayley's theorem.

11. Let R be a commutative ring with unit element and M is a ideal of R. Then prove that M is maximal ideal of R if and only if R/M is a field.

12. Let L is a finite dimension inner product space, then prove that V has an orthonormal set as a basis.

13. If L is a finite extension of K, K is a finite extension of F, then prove that L is a finite extension of F, and also prove that [L:K][K:F] = [L:F].

14. If V is a finite dimension over F, then prove that $T \in A(V)$ is singular if and only if there exists $v \neq 0$ in V such that $vT=0$.

15. V is a finite dimension over F, then for S, $T \in A(V)$ then prove the following
    (i) $r(ST) \leq r(T)$
    (ii) $r(ST) \leq r(S)$
    (iii) $r(ST) = r(TS) = r(T)$, for S regular in A(V).

16. If $T \in A(V)$ has all its characteristic roots if F, then prove that there is a basis of V is which the matrix of T is triangular.

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M.S.C. DEGREE EXAMINATION DECEMBER 2020.

Mathematics

Second Year

REAL ANALYSIS

Time : 3 Hours Maximum Marks : 75

SECTION-A (5 × 5 = 25 Marks)

Answer any FIVE questions.

1. Prove that every infinite subset of countable set is countable.

2. Prove that union of arbitrary collection of open set is open. Give an example for union of arbitrary collection of closed set is not closed.

3. Verify whether f'(0) exists for the following function

$$f(x) = \begin{cases} x\sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

4. Prove that a closed subset of a compact sets are closed .

5. Define gamma function and prove that $\Gamma(n+1) = n!$.

6. State and prove cauchy's criterion for uniform convergence .

7. State and prove weierstrass theorem.

8. Prove that a linear operator A in R^n is invertible if and only if $\det(A) \neq 0$.

PART – B (5 × 10 = 50 Marks)

Answer any FIVE questions.

9. If a set E in R^n, then prove that the following are equivalence
   (i) E is closed and bounded

(ii) E is compact

(iii) Every infinite subset of E has a limit point in E

10. Suppose $\{\{s_n\}$ and $\{\{t_n\}$ are sequences ,$\lim_{n\to\infty} s_n = s$ *and* $\lim_{n\to\infty} t_n = t$.
Then prove that (i) $\lim_{n\to\infty}(s_n + t_n) = s + t$
(ii) $\lim_{n\to\infty}(s_n\ t_n) = s\,t$

11. Prove that a mapping of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y .

12. Let f be a continuous mapping on a compact metric space X onto a metric space Y , then prove that f is uniformly continuous on X.

13. Derive Stirling's formula.

14. If f is a positive function on (0 ,∞) *such that*

(i) f(x+1) = xf(x)

(ii) f(1) =1

(iii) log f is convex

then prove that f(x) = $\Gamma(x)$.

15. State and Prove inverse function theorem.

16. Prove that $\lim\limits_{n\to\infty} (1 + 1\backslash n)^n = e$

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P.G. DIPLOMA EXAMINATION
DECEMBER 2020

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $\mathcal{B}$ is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y, then prove that the collection $\mathcal{D} = \{B \times C / B \in \mathcal{B}$ and $C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

2. If $f : X \to Y$; let X be metrizable, then prove that the function f is continuous if and only if for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$.

3. Prove that the image of a connected space under a continuous map is connected.

4. Show that every compact subset of a Hausdorff space is closed.

5. Define first and second countability axioms and prove that the product of two Lindelof spaces need not be Lindelof.

6. Define normed linear space and prove that $\left| \|x\| - \|y\| \right| \leq \|x - y\|$.

7. If M is a proper closed linear subspace of a Hilbert space H, then prove that there exists a non-zero vector z_0 in H such that $z_0 \perp M$.

8. If P is a projection on H with range M and null space N, then prove that $M \perp N \Leftrightarrow P$ is self-adjoint and in this $N = M^{\perp}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let $\bar{d}(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on R. If x and y are two points of R^w, define $D(x,y) = \text{lub}\left\{\frac{\bar{d}(x_i, y_i)}{i}\right\}$ then prove that D is a metric that induces the product topology
on R^w.

10. If $f : A \rightarrow X \times Y$ is given by the equation $f(a) = (f_1(a), f_2(a))$, then prove that f is continuous if and only if the functions $f_1 = A \rightarrow X$ and $f_2 : A \rightarrow Y$ are continuous.

11. Prove that the cartesian product of connected spaces is connected.

12. State and prove Urysohn lemma.

13. Prove that every regular space with a countable basis is normal.

14. State and prove Hahn-Banach theorem.

15. Let M be a closed linear subspace of a normed linear space N. If the norm of a coset $x + M$ in the quotient space $\frac{N}{M}$ is defined by $\| x + M \| = \inf \{ \| x + m \|, m \in M \}$ then prove that $\frac{N}{M}$ is a normed linear space. Further if N is a Banach space, then prove $\frac{N}{M}$ is also Banach.

16. If $\{e_i\}$ is an orthonormal set in a Hillbert space H and if x is an arbitrary vector in H, then show that $x - \Sigma(x, e_i)e_i \perp e_j$ for each j.

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