**PG-C-852 PGDAM - 11**

PG DIPLOMA EXAMINATION - DECEMBER 2020

# Applied Mathematics

First Year

## OPERATIONS RESEARCH

**Time: 3 Hours** **Maximum Marks: 75**

### Section A (5 x 5 = 25 Marks)

Answer any **FIVE** questions:

1. Define linear programming and mention its application in real life situation.

2. A company manufactures two types of products namely A and B and sells them at a profit of Rs. 2 on type A and Rs.3 on type B. Each product is processed on two machines $M_1$ and $M_2$. Type A requires 2 minutes of processing time on $M_1$ and 3 minutes on $M_2$. Type B requires 2 minutes of processing time on $M_1$ and 4 minutes on $M_2$. Machine $M_1$ is available for not more than 6 hours 40 minutes while machine$M_2$ is available for 10 hours during any working day. Only formulate the problem as a LPP so as to maximize the profit.

3. Explain briefly the probabilistic dynamic programming.

4. Explain the maximum flow problem in network analysis.

5. Solve the following game graphically

| $B_1$ | $B_2$ | $B_3$ |
|---|---|---|
| 2 | – 1 | – 2 |
| – 2 | 4 | 1 |

6. Explain the procedure for solving IPP using cutting plane method.

7. Derive the formula to find the expected number of customers in the system in (M/M/1) : ($\infty$ /FCFS) model.

8. Explain unconstrained optimization in non-linear programming.

## Section B (5 x 10 = 50 Marks)

Answer any **FIVE** questions.

9. Solve the LPP using dual simplex method, Minimize $Z = 2x_1 + x_2$ subject to the constraints $3x_1 + x_2 \geq 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \geq 3$ and $x_1, x_2 \geq 0$

10. (i) Explain parametric linear programming. How does it differ from sensitivity analysis?

    (ii) What is sensitivity analysis? Discuss the effect of variation of $b_i$.

11. Use dynamic programming to solve the following LPP:
    Maximize $Z = 3x_1 + 5x_2$ subject to $x_1 \leq 4$, $x_2 \leq 6$, $3x_1 + 2x_2 \leq 18$ and $x_1, x_2 \geq 0$

12. From the given data, draw a network and identify the critical path and its duration.

| Activity | 0-1 | 1-2 | 1-3 | 2-4 | 2-5 | 3-4 | 3-6 | 4-7 | 5-7 | 6-7 |
|---|---|---|---|---|---|---|---|---|---|---|
| Duration (in days) | 2 | 8 | 10 | 6 | 3 | 3 | 7 | 5 | 2 | 8 |

13. Solve the following game using linear programming method:

Player B

| | | | | |
|---|---|---|---|---|
| | 9 | 1 | 4 | |
| Player | 0 | 6 | 3 | A |
| | 5 | 2 | 8 | |

14. Use Branch and Bound method to solve the following IPP: Maximize $Z = 2x_1 + 2x_2$ subject to $5x_1 + 3x_2 \leq 8$, $x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$ and are integers.

15. Cars arrive at a petrol pump, having one petrol unit, in Poisson fashion with an average of 10 cars per hour. The service time is distributed exponentially with a mean of 3 minutes. Find
    (i) the average number of cars in the system
    (ii) average waiting time in the queue.
    (iii) average queue length.
    (iv) the probability that the number of cars in the system is 2.

16. Explain geometric programming of solving non linear programming problem and mention its advantages.

# P.G. DIPLOMA EXAMINATION DECEMBER 2020

## Applied Mathematics

## MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. A random variable X has the following the probability function :

| Values of $X$ : | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $p(x)$ : | 0.1 | $K$ | 0.2 | $2K$ | 0.3 | $K$ |

   (a) Find the value of K and calculate mean and variance.

   (b) Construct the c.d.f. $F(x)$.

2. If X and Y are random variables then prove that $E(X+Y)=E(X)+E(Y)$.

3. Let $X_1, X_2, ....$ be i.i.d. Poisson variates with parameter $\lambda$. Use central limit theorem to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + .... + X_n$; $\lambda = 2$, and $n = 75$.

4. Define an unbiased estimate. If T is an unbiased estimator of $\theta$, then show that $T^2$ is a biased estimator of $\theta^2$.

5. In a sample of 1000 people in Kerala, 540 are tea consumers and the rest of coffee consumers. Can we assume that both tea and coffee are equally popular in Kerala at 1% level of significance?

6. Find the M.L.E. for the parameter $\lambda$ of a Poisson distribution.

7. Let $x_1, x_2 .....x_n$ be a random sample from $N(\mu, \sigma^2)$ population. Find sufficient estimators for $\mu$ and $\sigma^2$.

8. Prove that in sampling from a $N(\mu, \sigma^2)$ population the sample mean is a consistent estimator for $\mu$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. State and prove the Chebychev's inequaltiy.

10. In a continuous distribution whose relative frequency density is given by

$$f(x) = y_0.x(2-x) \; 0 \le x \le 2$$

Show that the distribution is symmetrical.

11. (a) Find the m.g.f. of normal distribution.

(b) State the Lindbery – Levy form of central limit theorem. (7 + 3)

12. (a) Obtain the 95% and 99% confidence interval for $\mu_1$ where $n$ is large.

(b) If $x_1, x_2, ....x_n$ is a random sample from a normal population $N(\mu, 1)$, then show that estimator of $\mu^2 + 1$.

13. In a random sampling from normal population $N(\mu, \sigma^2)$ find the maximum likelihood estimators for

(a) $\mu$ when $\sigma^2$ is known

(b) $\sigma^2$ when $\mu$ is known.

14. State and prove the Neymann – Pearson Lemma.

15. Obtain $100(1-\alpha)\%$ confidence limits (for large samples) for the parameter $\lambda$ of the Poisson distribution.

16. State and prove the Cramer – Rao inequaltiy.

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