UG-C-438	BMS-11/BMC–11

B.Sc. DEGREE EXAMINATION — DECEMBER 2020

Mathematics/Mathematics with
Computer Applications

First Year

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 75**

SECTION- A **(5 × 5 = 25 Marks)**

Answer any FIVE questions.

1. If $y = x^2 \cos x$, prove that $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + (x^2 + 6)y = 0$.

2. If $v = e^{a\theta} \cos(a \log r)$ show that $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v}{\partial^2} = 0$.

3. Find the maximum value of $\frac{\log x}{x}$ for $x > 0$.

4. Find the envelope of the family of circles $(x - a)^2 + y^2 = 2a$, where a is the parameter.

5. Show that : $\int_0^{\pi} \frac{dx}{5 + 3\cos x} = \frac{\pi}{4}$.

6. Evaluate : $\int_0^3 \int_1^2 xy(x + y)dy\, dx$.

7. If $\sum V_n$ is convergent and $\frac{u_n}{v_n}$ tends to a limit other than zero as $n \to \infty$, prove that $\sum u_n$ is convergent.

8. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$.

SECTION - B (5 × 10 = 50 Marks)

Answer any FIVE questions.

9. If $y = \sin^{-1} x$, prove that $(1 - x^2)y_2 - xy_1 = 0$ and $(1 - x^2)y_{n+2} - (2n - 1)xy_{n+1} - n^2 y_n = 0$.

10. Find the maxima and minima of the function $2(x^2 - y^2) - x^4 + y^4$.

11. Find radius of curvature at the point 't' of the curve $x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t)$.

12. Evaluate : $\int_0^{\pi/2} \log \sin x dx$.

13. Find a reduction formula for $\int \tan^n x \, dx$, n is a integer. Hence find $\int \tan^4 x \, dx$.

14. Find $\iint xy \; dx \; dy$ taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

15. Test for convergency and divergency of the series $1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^2 x^3}{4!} + ...$

16. Discuss the convergency of the series $\frac{1}{1 + x} + \frac{1}{1 + 2x^2} + \frac{1}{1 + 3x^3} + ...$ for positive values of x .

UG-C-440 **BMS-11**

B.Sc Degree Examination - December 2020

Mathematics

First Year

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 4 = 20 Marks)**

Answer any **FIVE** questions:

1. Find the maximum value of $\sin^2 x\,(1 + \cos x)^3$
2. Transform $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ into polar coordinates.
3. Find the envelope of the family of straight lines $y + tx = 2at + at^3$ the parameter being t.
4. If $u_n = \int_0^{\pi/2} x^n \cos x dx$ show that $u_n + n(n-1)\, u_n = \left(\frac{\pi}{2}\right)^n$
5. Change the order of integration in $\int_0^a \int_{\frac{x^2}{2}}^{2a-x} xydxdy$
6. State and prove Cauchy's general principle of convergence of a sequence of real numbers.
7. Discuss the convergence or divergence of
$\frac{1^2}{2^2} + \frac{1^2-3^2}{2^2-4^2} + \frac{1^2-3^2-5^2}{2^2-4^2-6^2} + \dots$
8. State and prove Cauchy's Root test.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** questions.

1. If y = acos(logx) + bsin(logx) prove that $x^2y_{n+2} + (2n + 1)xy_{n+1}+(n^2+1)y_n=0$.

2. Find the extreme values of $y^2 + 4xy +3x^2 + x^3$

3. a. Show that the evolutes of the cycloid x = a(φ – sinφ), y = a(1 – cosφ) is another cycloid.
   b. Find the (p – r) equation of the curve r = asinφ.

4. Find the area of the cardioids r = a (1 + cosφ) and the volume of solid generated by this cardioids when it revolves about the initial line.

5. Using triple integrals, find the volume of the portion of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which lies in the first octant.

6. a) Prove that the harmonic series $\sum_{n=1}^{\infty} 1/n^p$ is convergent if p > 1 and divergent if p $\leq$ 1.
   b) Test for convergence the series $\frac{2}{1^p} + \frac{3}{2^p} + \frac{4}{3^p} + \dots\dots\infty$.

7. a) Prove Cauchy's condensation test.
   b) Show the $\sum u_n$ where $u_n = \frac{1}{n(logn)^p}$ is convergent when p > 1 and divergent when p $\leq$ 1.

8. State and prove D'Alembert's ratio test.

U.G. DEGREE EXAMINATION – DECEMBER 2020

First Year

Mathematics

TRIGNOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Express $\cos 6\theta$ interms of $\cos\theta$.

2. Prove that $\cos h^{-1} x = \log_e \left[x + \sqrt{x^2 - 1} \right]$.

3. Prove that the planes $x + 2y + 2z = 0$, $2x + y - 2z = 0$ are right angles.

4. Find the equation of the plane parallel to $2x - 3y + 5z + 12 = 0$ and passing through the points (2, 3, 1).

5. Find centre and radius of the sphere $16x^2 + 16y^2 + 16z^2 - 16x - 8y - 16z - 55 = 0$.

6. Find the equation of the sphere with centre (1, –1, 2) and touching the plane $2x - 2y + z = 3$.

7. If $\phi = x^2 + y^2 - z - 1$ find $grad\ \phi$ at (1, 0, 0).

8. If $\vec{F} = x^2\vec{i} + xy\vec{j}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ from (0, 0) to (1, 1) along the line $y = x$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Prove that $\cos^8 \theta = \frac{1}{2^7}[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$.

10. Find $\log(1+i)$.

11. Find the equation of the plane passes through the intersection of the planes $2x + 3y + 10z - 8 = 0$ $2x - 3y + 7z - 2 = 0$ and is perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

12. Find the image of the point (2, 3, 5) in the plane $2x + y - z + 2 = 0$.

13. Obtain the equation of the plane passing through the points (2, 2, –1), (3, 4, 2) and (7, 0, 6).

14. Find the shortest distance between the lines
$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-15}{-5}.$$

15. Find the equation of the sphere which pass through the circle $x^2+y^2+z^2=5,\ x+2y+3z=5$ and touch the plane $4x+3y=15$.

16. Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ of the vector point function $\vec{F}=xz^3\vec{i}-2x^2yz\vec{j}+2yz^4\vec{k}$ at the point (1, –1, 1).

UG-C-441 **BMS-12**

B.Sc Degree Examination - December 2020

Mathematics

First Year

TRIGONOMETRY, ANALYTICAL GEOMETRY (3D) & VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 70**

Section - A **(5 x 4 = 20 Marks)**

Answer any **FIVE** questions:

1. Prove that $\frac{\cos 5\,\theta}{\cos 5\,\theta} = 1 - 12\,sin^2\theta + 16\,sin^4\theta.$

2. Determine a and b so that $It_{\varphi \to 0} \left(\frac{\text{a } - \theta\sin\theta - bcos\theta}{\theta^4}\right) = \frac{1}{12}$

3. If tanh $\frac{u}{2}$ = tan $\frac{\theta}{2}$, prove that u = log ≤ $(\frac{\pi}{4} + \frac{\theta}{2})$

4. Find the equation of the plane through the points (2,2,1) and (0,3,6) and perpendicular to the plane 2x+6y+6z = 9.

5. Find the angle between the planes x – y + 2z – 9 = 0 and 2x + y + z = 7.

6. Find the equation of the straight line through (3,2,-8) and perpendicular to –3x + y + 2z – 2 = 0.

7. If $\vec{r} = acos\omega t + bsin\omega t$, where a,b, ω are constants. Prove that $\vec{r} = \frac{d\vec{r}}{dt} = \omega a$ x b and $\frac{d^2\vec{r}}{dt^2} + \omega^2\vec{r} = 0$.

8. If $\vec{F} = x^2y\vec{\imath} + y^2z\vec{\jmath} + z^2x\vec{k}$, find curl $\vec{F}$ and curl curl $\vec{F}$.

Section - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** questions.

1. Express $\sin^8\theta$ in a series of cosines of multiple of θ.

2. (a) If $\frac{\cos 5\theta}{\theta} = \frac{5045}{5046}$, show $\theta = 1^o 58'$approximately.
   (b) If cos (x + iy) = $\cos\theta + i sin\theta$, prove that cos2x + cosh2y = 2

3. a. Prove that log $\left[\frac{(1+i)\,(2+i)}{3+i}\right]$ is purely imaginary.
   b. Sum to infinity the series $\frac{\sin a}{3} + \frac{\sin 2a}{3^2} + \frac{\sin 3a}{3^3} + \ldots$

4. a. Show that point (0, -1,0), (2,1,-1) (1,1,1) and (3,3,0) are coplanar and find the equation of the plane on which they lie.
   b. Find the image of the point (1,2,-3) in the plan 3x – 3y + 10z = 26

5. a. Prove that the lines

$$\frac{X-4}{1} = \frac{Y+3}{-4} = \frac{Z+1}{7} \; and \; \frac{X-1}{2} = \frac{Y+1}{3} = \frac{Z+10}{8}$$

   intersect and find the coordinates of their point of intersection.
   b. Find the shortest distance between the lines

$$\frac{X-3}{3} = \frac{Y-8}{-1} = \frac{Z-3}{1};\ \frac{X+3}{-3} = \frac{Y+7}{2} = \frac{Z-6}{4}$$

6. a) Find the equation of the sphere passing through (1,-3,4), (1,-5,2), (1,-3,0) and having its center on the plan x + y + z = 0.
   b) Find the equation of the cone whose vertex is the point (1,1,0) and whose base is the curve y = 0, $x^2 + z^2 = 4$

7. a) Prove that curl (φu) = ∇φ x u + φ curl u
   b) Show that $r^n \vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if n = 3.

8. a) Show, by applying Green's theorem that the area bounded by a simple closed curve C is = ½ $_C\int\ (xdy - ydx)$ and hence find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

UG-C-442 **BMS–13**

B.Sc. DEGREE EXAMINATION – DECEMBER 2020

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve $(x^2 + y^2 + 2x)dx + 2y\,dy = 0$.

2. Solve $(3D^2 + D - 14)y = 13e^{2x} + e^{-x}$.

3. Solve $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

4. Solve $y - 2px = x^2 p^4$.

5. Eliminate the constants a and b from $ax^2 + by^2 + z^2 = 1$.

6. Solve $xzp + yzq = xy$.

7. If $f(t) = t^2, 0 < t < 2$ and $f(t+2) = f(t)$ Find $L[f(t)]$.

8. Find $L^{-1}\left[\dfrac{5s+3}{(s-1)(s^2+2s+5)}\right]$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Solve $p^2 + 2py\cot x = y^2$.

10. Solve $(1+2x)^2 \dfrac{d^2y}{dx^2} + (1+2x)\dfrac{dy}{dx} + y = 8(1+2x)^3$.

11. Apply the method of variation of parameters to solve $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = x^2 e^x$.

12. Solve $(x^3 - x)\dfrac{d^3y}{dx^3} + (8x^2 - 3)\dfrac{d^2y}{dx^2} + 14x\dfrac{dy}{dx} + 4y = \dfrac{2}{x^3}$.

13. Find the equation of the cone satisfying the equation $xp + yq = z$ and passing through the circle $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$.

14. Find the complete integral of the equation :

$$p^2 + q^2 - 2px - 2qy + 2xy = 0 .$$

15. (a) If $L[f(t)] = F(s)$ then prove that

$$L\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n}\{F(s)\} \text{ for } n = 1, 2, 3 \ldots$$

(b) Find $L^{-1}\left[\dfrac{1}{s(s+2)^3}\right]$.

16. Solve the simultaneous equations

$$\frac{dx}{dt} - \frac{dy}{dt} - 2x + 2y = 1 - 2t$$

$$\frac{d^2x}{dt^2} + 2\frac{dy}{dt} + x = 0$$

given that $x = 0$, $y = 0$, $\dfrac{dx}{dt} = 0$ when $t = 0$.

UG-C-670 BMS-13

U.G. DEGREE EXAMINATIONS

DECEMBER -2020

Mathematics

First Year

DIFFERENTIAL EQUATIONS

Time: 3 Hours **Maximum Marks: 70**

PART – A **(5 X 4 = 20 Marks)**

Answer any five questions.

1. Solve: $p^2 - p(e^x + e^{-x}) + 1 = 0$
2. Solve: $(x^4 + y^4)\,dx - xy^3\,dy = 0$
3. Solve: $(D^2 - 9)y = 6e^{3x} + x\,e^{3x}$
4. Solve: $\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y+2x)}$
5. Eliminate the arbitrary functions f and g from $z = f(x) + e^y.g(x)$
6. Solve: $p(1 + q^2) = q(z - 1)$
7. Find: $L[te^{-1}\sin t]$
8. Find L-1 $[\frac{s^2}{(s-1)^4}]$

PART – B **(5 X 10 = 50 Marks)**

Answer any five questions.

1. Solve: $\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$
2. Solve: $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = y = \frac{\log x \sin(\log x) + 1}{x}$
3. Solve the equation $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$ by the method of variation of parameters.
4. Solve: $(1+x+x^2)\frac{d^2y}{dx^2} - (3+6x)\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 0$
5. Find the complete integral of the equation
   $p^2 + q^2 - 2px - 2qy + 2xy = 0$
6. Sole: $z2(p^2+q^2) = x^2 + y^2$
7. (i) Find L [f (t)] where $f(t) = \sin t$ when $0 < t < \pi$
   $= 0$ when $\pi < t < 2\pi$
   and $f(t + 2\pi) = f(t)$
8. Using Laplace transforms, solve the equation

$$t\frac{d^2y}{dt^2} - (2+t)\frac{dy}{dt} + 3y = t - 1 \text{ when } y(0) = 0$$

UG-C-443	BMS-21

B.SC DEGREE EXAMINATION - DECEMBER 2020

MATHEMATICS

SECOND YEAR

GROUPS AND RINGS

Time: 3 Hours **Maximum Marks: 75**

PART- A **(5 x 5 = 25 Marks)**

Answer any FIVE questions:

1. Let Z denote the set of integers and Z^* denote the set of non-zero integers. On $S = Z \times Z^*$ define a relation R by □$(a,b)\ R\ (c,d)$ if $ad = bc$. Prove that R is an equivalence relation on S. Also state what each equivalence represent?

2. Define a cyclic group and prove that a subgroup of a cyclic group is cyclic.

3. Define centre of a group G and prove that it is a normal subgroup of G.

4. Define the following and give examples:
   (a) Integral domain
   (b) Field

5. Also prove that any field is an integral domain.
   R = $\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} / a, b \epsilon R\right\}$ and let C be the ring of complex numbers. Show that the map $\varphi : R \to C$ given by φ (a + ib) = $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

6. Define Prime and maximal ideals of a ring and prove that any maximal ideal of a ring is a prime ideal of the ring.

7. Prove that a finite integral domain is a field.

8. Define a Euclidean ring and prove that any field is a Euclidean ring.

9. Find the field of fractions of the ring of Gaussian integers.

PART-B **(5 x 10 = 50 Marks)**

Answer any FIVE questions.

10. Let R be an equivalence relation on a set S. Prove that the set of all equivalence classes under R is a partition of S.

11. Prove that any finite group is isomorphic to a subgroup of $A(S)$, for some appropriate S.

12. Let G be a group and let N be a normal subgroup of G. Define a binary operation on the set G / N of all right cosets of N in G to make G / N into a group. Also write down the order of G / H.

13. Prove that any Euclidean ring is a Principal ideal domain.

14. Let R be a commutative ring with identity. Show that an ideal M of R is maximal if R / M is a field.

15. Prove that the ring of Gaussian integers is Euclidean ring.

16. Prove that the ring, F [x] of polynomial, in X over a field F is Euclidean ring.

17. Let R be an equivalence relation on a set S. Prove that the set of all set of all equivalence classes under R is a partition of S.

18. Prove that any finite group is isomorphic to a subgroup of $A(S)$, for some appropriate S.

UG-C-444 BMS-22

B.Sc. DEGREE EXAMINATION – DECEMBER, 2020.

B.SC., MATHEMATICS

SECOND YEAR

STATISTICS AND MECHANICS

Time: 3 Hours **Maximum Marks : 75**

PART – A **(5 X 5 = 25 Marks)**

Answer any FIVE questions.

1. The average salary of male employees in a firm was Rs. 5,200 and that of females was Rs. 4,200. The mean salary of all the employees was Rs. 5,000. Find the percentage of male and female employees.

2. If arithmetic mean and geometric mean of two values are 10 and 8 respectively. Find values.

3. Distinguish between correlation and regression analysis.

4. Estimate U_2 from the following table :

x :	1	2	3	4	5
U_x :	7	–	13	21	37

5. What is the chance that a leap year selected at random will contain 53 Sundays?

6. Find the mean and variance of Poisson distribution.

7. Prove that $gT^2 = 2R\tan\alpha$, where T is the time of flight, R the horizontal range and α the angle of projection of a particle projected from the ground.

8. A particle moving with S.H.M. from an extremity of the path towards the centre is observed to be at distance x_1, x_2, x_3 from the centre at the ends of 3 consecutive seconds. Show that the time of a complete oscillation is $\dfrac{2\pi}{\cos^{-1}\left(\dfrac{x_1+x_3}{2x_2}\right)}$.

PART – B **(5 X 10 = 50 Marks)**

Answer any FIVE questions.

1. Find the coefficient of sleekness from the following information.

Difference of the Quartiles = 8

Mode = 11

Sum of two Quartiles = 22

Mean = 8.

2. Ten participates were ranked according to their performance in a musical test by the 3 judges in the following data :

	1	2	3	4	5	6	7	8	9	10
Rank by X :	1	6	5	10	3	2	4	9	7	8
Rank by Y :	3	5	8	4	7	10	2	1	6	9
Rank by Z :	6	4	9	8	1	2	3	10	5	7

Using rank co-orelation method, discuss which pair of judges has the nearest approach to common linkages of music.

3. Find $f(8)$ by Lagrange's formula from the data :

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

4. A bag contains 5 red and 3 green balls and a second bag contains 4 red and 5 green balls. One of the bags is selected at random and a draw of 2 balls is made from it. What is the probability that one of them is red and the other is green?

5. A manufacturer of television sets known's that of an average 5% of his product is defective. He takes television in consignment of 100 and guarantees that not more than 4 sets will be defective. What is the probability that a television set will fail to meet the guaranteed quality.

6. The following table gives the yields of 15 samples of plot under three varieties of seed.

A	B	C
20	18	25
21	20	28
23	17	22
16	15	28
20	25	32

Test using analysis of variance whether there is a significant difference in the average of yield of seeds.

7. A ball *A* impinges directly on an exactly equal and similar ball B lying on a smooth horizontal l table. If e is the coefficient of restitution, prove that after impact the velocity of B is to that of A as $1 + e : 1 - e$.

8. Derive the differential equation of the central orbit.

UG-C-445	BMS-23

B.Sc. DEGREE EXAMINATION – DECEMBER 2020

Second Year

Mathematics

CLASSICAL ALGEBRA AND NUMERICAL METHODS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Give the expansion for (a) $(1+x)^{-2}$ (b) $(1-x)^{-1}$ (c) $(1-x)^{-n}$.

2. Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in AP.

3. State (a) Rolle's theorem. (b) Sturm's theorem.

4. Use forward differences to find $f(x)$ when $x = 4$:

x :	3	5	7	9
$f(x)$:	180	150	120	90

5. Compute $\int_0^1 \frac{1}{1+x^2}$ with $h = 0.5$ using trapezoidal rule.

6. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$ given that $1+i$ is a root.

7. Find a root of $x^3 - 4x - 9 = 0$ that lies between 2 and 3 using bisection method.

8. Construct Newton's forward interpolation polynomial for the following data

x :	4	6	8	10
y :	1	3	8	16

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Sum to infinity the series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \ldots\infty$.

10. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

11. Remove the second term from the equation $x^3 - 6x^2 + 11x - 6 = 0$.

12. Find the root of $x^3 - 6x + 4 = 0$ that lies between 0 and 1 using Newton–Raphson method correct to four decimal places.

13. Given

x :	0	4	8	12
y :	143	158	177	199

Calculate y_5 by Bessel's formula.

14. Find $f(8)$ using Newton's divided difference formula.

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

15. Find $f'(5)$ using Newton's formula

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

16. Apply the fourth order Rung-kutta method to find an approximate value of y when $x = 0.2$ given $y' = x + y,\ y(0) = 1$.

UG-C-446 BMS-31

B.Sc Degree Examination - December 2020

Mathematics

Third Year

REAL AND COMPLEX ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

Section- A **(5 x 5 = 25 Marks)**

Answer any **FIVE** questions:

1. Prove that Q is the set of all rational numbers in countable.

2. Let (x_n) be a convergent sequence is a metric space (x,d). Then prove that
   (i) The limit is unique.
   (ii) (x_n) converges to x if and only if d (x_n, x) tends to 0.

3. Let f be a function from a metric space X to a metric space Y. Then prove that f is continuous on X if and only if f^{-1} (G) is open in X whenever G is open in Y.

4. State and prove Heine Borel theorem.

5. If f and g are real valued differentiable functions defined on R' then prove that f/g is differentiable at all points where $g \neq 0$.

6. Explain why Riemann sphere is a better model for the extended complex number system than the plane.

7. Discuss the differentiability of the function f |z| = |z| on C.

8. If C is the circle with centre z_0 and radius r then finds

$$\int \frac{dz}{(z - z_0)^{m+1}}$$

Section- B **(5 x 10 = 50 Marks)**

Answer any **FIVE** questions.

9. On R x R prove that the metrics d_1, d_2, d_3, defined below are equivalent:
$d_1((x_1, x_2), (y_1, y_2,)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
$d_2((x_1, x_2), (y_1, y_2,)) = \max\{|x_1 - y_1|, |x_2 - y_2,|\}$
$d_3((x_1, x_2), (y_1, y_2,)) = \{|x_1 - y_1| + |x_2 - y_2,|$

10. State and proved the cantor's intersection theorem. Prove that the condition "F_n is closed" cannot be removed from the statement of the theorem.

11. Let X be a metric space. Then prove that the following are equivalent:
    i. X is compact
    ii. Any infinite subset of X has a limit point in X.
    iii. X is sequentially compact.
    iv. X is totally bounded and complete.
    v.

12. State and prove the Taylor's theorem with Cauchy form of remainder.

13. Write an essay on elementary bilinear transformations

14. i. Find an analytic function whose imaginary part is 2xy + 1.
ii. Find a bilinear transformation which maps the inside of the unit circle onto the upper half plane.

15. State and prove Cauchy integral theorem for rectangle.

16. i. State and prove Cauchy's reside theorem.
ii. Show that $\int_0^\infty \frac{bcosax}{x^2+b^2} dx = \int_0^\infty \frac{xsinax}{x^2+b^2} dx = \frac{\pi}{2} e^{-ab}$ (a,b > 0).

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B.Sc Degree Examination - December 2020

Mathematics

Third Year

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

PART -A **(5 x 5 = 25 Marks)**

Answer any FIVE questions:

1. Prove that the intersection of two sub-spaces of a vector space is a subspace.

2. Define linearly independent set and linearly dependent set.

3. Show that dim V = rank T + nullity T, with usual notations.

4. Obtain the matrix representing the linear transformation $T : V_3 \rightarrow V_3(R)$ given by $T(a,b,c) = (3a, a - b, 2a + b + c)$ w.r.t. the standard basis $\{e_1,e_2,e_3\}$.

5. State and prove Schwartz's inequality.

6. Define a bilinear form and quadratic form.

7. Reduce the quadratic form $x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2$ to the diagonal form.

8. Prove that every chain is a lattice.

PART- B **(5 x 10 = 50 Marks)**

Answer any FIVE questions.

1. State and prove the fundamental theorem of homomorphism on vector spaces.

2. Prove that any two bases of a finite dimensional vector space V have the same number of elements.

3. State and prove any three equivalent conditions for a set to be a basis for a vector spaces.

4. Explain Gram-Schmidt orthogonalisation process.

5. Prove that V = w $\oplus$ $w^{\perp}$, where W is a subspace of a finite dimensional inner product space V.

6. Let f be a symmetric bilinear form defined on V. Let q be the associated quadratic form. Prove that,
   (i) f (u,v) = $1/4${q(u+v) – q (u – v)}
   (ii) f (u,v) = $1/2$ {q(u+v) – q(u) – q(v)}

7. Prove that the set of all normal subgroups of any group is modular lattice.

8. (a) In a Boolean algebra if a v x = b v x and a v x 1 = b v x 1, then prove that a = b.

   (b) Show that in a Boolean algebra [a v (a' ^ b)] 6 [b v (b ^ c)] = b.

UG-C-446	**BMS–33**

B.Sc. DEGREE EXAMINATION – DECEMBER - 2020

Mathematics

Third Year

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time: 3 Hours Maximum Marks: 75

SECTION A ($5 \times 5 = 25$ Marks)

Answer any FIVE of the following.

1. Write down the following LPP in standard form:

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$

   Subject to the constraints

$$2x_1 - 3x_2 \leq 3,$$
$$x_1 + 2x_2 + 3x_3 \geq 5$$
$$3x_1 + 2x_3 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0.$$

2. Prove that the dual of the dual is the primal.

3. Explain the difference between a transportation problem and an assignment problem.

4. Obtain an initial basic feasible solution to the following transportation problem using the north-west corner rule.

   D E F G Available

A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	200	

5. For the game with payoff matrix.

$$\begin{array}{cc} & \text{Player A} \\ \text{Player B} & \begin{pmatrix} -1 & 2 & -2 \\ 6 & 4 & -6 \end{pmatrix} \end{array}$$

Determine the best strategies for Player A and B and also the value of the game for them. In this game (a) fair, (b) strictly determinable.

6. Solve the following game and determine the value of the game

$$\begin{array}{cc} & \text{B} \\ \text{A} & \begin{pmatrix} 4 & -4 \\ -4 & -4 \end{pmatrix} \end{array}$$

7. Explain:

   (a) Set-up cost

   (b) Carrying cost

   (c) Shortage cost

8. A car park contains 5 cars. The arrival of cars is Poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is exponential distribution with mean of 5 hours. How many cars are in the car park on an average?

SECTION B (5 × 10 = 50 Marks)

Answer any FIVE of the following.

9. Use simplex method to solve

Maximixe $Z = 5x_1 + 3x_2$

Subject to the constraints

$$x_1 + x_2 \le 2,$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_3 \le 12$$
$$x_1, x_2 \ge 0.$$

10. Use penalty (Big–M) Method to solve

Maximixe $Z = 2x_1 + 3x_2$

Subject to the constraints

$$x_1 + 2x_2 \le 4$$
$$x_1 + x_2 = 3 ; x_1 \ge 0 \text{ and } x_2 \ge 0.$$

11. A company has 4 machines to do 3 Jobs. Each Job can be assigned to one and only one machine. The cost of each Job on each machine is given below. Determine the Job assignments which will minimize the total cost.

$$\begin{array}{cc} & \text{Machine} \\ & \begin{array}{cccc} \text{W} & \text{X} & \text{Y} & \text{Z} \end{array} \\ \text{Job} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \end{array} & \begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 18 \\ 10 & 15 & 19 & 22 \end{pmatrix} \end{array}$$

12. Use Vogel's Approximation method to obtain an initial basic feasible solution of the transportation problem.

$$\begin{array}{c|cccc|c} & D & E & F & G & \text{Available} \\ \hline A & 11 & 13 & 17 & 14 & 250 \\ B & 16 & 18 & 14 & 10 & 300 \\ C & 21 & 24 & 13 & 10 & 400 \\ \hline \text{Demand} & 200 & 225 & 275 & 250 & \end{array}$$

13. Solve the following 2×3 game graphically

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{pmatrix} \end{array}$$

14. Using the principle to dominance, solve the following game.

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix} \end{array}$$

15. Derive the formula for optimum lot size for a manufacturing model with no shortages, uniform demand and infinite production rate.

16. Explain the queuing mode (M/M/1): (∞/FIFO). Obtain its steady state solution.

UG-C-449	BMS-33N

B.SC DEGREE EXAMINATION - DECEMBER 2020

MATHEMATICS

THIRD YEAR

OPTIMIZATION TECHNIQUES

Time: 3 Hours **Maximum Marks: 75**

Section - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** questions:

1. Determine an initial basic solution to the following transportation problem by using the north-west corner method:

	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	2	5
	6	10	15	4	35

2. Solve the following 2 x 4 game graphically.

Player B

Player A		B_1	B_2	B_3	B_4
	A_1	2	1	0	-2
	A_2	1	0	3	2

3. A super market has two girls ringing up sales at the counters. If the service time for each counter is exponential with mean 4 minutes, and if people arrive in a Poisson fashion at the rate of 10 an hour,
   (a) What is the probability of having to wait for service?
   (b) What is the expected percentage of idle time for each girl?

(c) Find the average queue length as the average number of units in the system.

4. Use Charne's penalty method to
Minimize $Z = 2x_1 + x_2$
Subject to the constraints;
$3x_1 + x_2 = 3$
$4x_1 + 3x_2 = 3$
$x_1 + 2x_2 \geq 6$
$x_1 + 2x_2 \leq 3$
$x_1, x_2 > 0$

5. Use simplex method to solve the following L.P.P
Maximize $Z = 7x_1 + 5x_2$
Subject to the constrains:
$x_1 + 2x_2 \leq 6$
$4x_1 + 3x_2 \leq 12$
$x_1, x_2 \geq 0$

6. Use the duality to solve the following L.P.P.:
Maximize $Z = 7x_1 + 5x_2$
Subject to the constrains:
$x_1 + 2x_2 \leq 10$
$x_1 + x_2 \leq 6$
$x_1 - x_2 \leq 1$
$x_1, x_2, x_3 \geq 0$

7. Find the optimum integer solution to the following L.P.P:
Maximize $Z = x_1 + 2x_2$
Subject to the constrains:
$x_1 + x_2 \leq 7$
$2x_1 \leq 11$
$2x_2 \leq 7$
$x_1, x_2 \geq 0$ and are integers.

8. Maximize $Z = \frac{3}{4}x_1 - 150x_2 + \frac{1}{60}x_3 - 6x_4$
Subject to the constraints:
$\frac{1}{4}x_1 - 60x_2 - \frac{1}{26}x_3 + 9x_4 \leq 0$
$\frac{1}{4}x_1 - 90x_2 - \frac{1}{60}x_3 + 3x_4 \leq 0$
$x_3 \leq 1$
$x_1, x_2, x_3, x_4, \geq 0$

Section- B **(5 x 10 = 50 Marks)**

Answer any **FIVE** questions.

9. Use dual simplex method to solve the L.P.P

   Maximize $Z = x_1 + 2x_2 + 3x3$

   Subject to the constrains:

   $x_1 - x_2 + 2x3 \geq 4$

   $x_1 - x_2 + 2x3 \leq 8$

   $4x_1 + 3x_2 \geq 2$

   $x_1, x_2, x3 \geq 0$

10. Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

			Job		
Persons	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule

11. Obtain an initial basic feasible solution to the following T.P. using the Vogel's approximation method:

			Stores		
Warehouses	I	II	III	IV	Availability
A	5	1	3	3	34
B	3	3	5	4	15
C	6	4	4	3	12
D	4	-1	4	2	19
Requirement	21	25	17	17	80

12. Solve the following 3 x 3 game by linear programming:

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix} \end{array}$$

13. Explain EOQ model without Shortage.

14. Explain (M/M/1): (∞/FIFO) model

UG-C-450	BMS-34

B.Sc. DEGREE EXAMINATION – DECEMBER 2020

MATHEMATICS

THIRD YEAR

PROGRAMMING IN C AND C++

Time: 3 Hours **Maximum Marks: 75**

PART A (5X5=25 Marks)

Answer any FIVE questions.

1. Explain the various input statements in C.
2. Write a C program to read three numbers and print the largest among the three numbers.
3. Explain the switch statement with example.
4. Write a C program to read the marks of the students in a class and print the average of the marks using for loop.
5. Explain (i) pointer declaration (ii) pointer operator (iii) pointer expression (iv) pointer comparison (v) pointer to array.
6. Write a C function to calculate the factorial of a number and use it in the main to calculate nCr. (nCr=n!/(r!.n-r!)).
7. Explain call by reference and return by reference with example.
8. Write a C++ program to create a class with field members name, age and blood group. Read the data of employees in a institution and print all those name with age greater than 20 and with blood group O+ve.

PART B (5X10=50 Marks)

Answer any FIVE questions.

9. Explain the various operators available in C with example.
10. Explain the for loop statement in C with example.
11. Write a program to read a quadratic equation and print their roots (Using function).

12. Write C functions (i) to read a m X n matrix (ii) to print a m X n matrix (iii) to find the sum of two matrix. Use it in the main program to read two matrix and to print their sum if possible.

13. Write a C program to create a structure with field members name of an item, item code, cost of each item and quantity in stock. Read the data and update the data using a function and calculate the total value using another function print the output.

14. Explain (i) fopen (ii) fclose (iii) fseek (iv) fprintf (v) fscanf.

15. Write a C++ program to read two complex numbers and print their sum using operator overloading.

16. Develop an object oriented program to create a library information system containing the following for all books in the library.

    Accession Number, Name of the author, Title of the book, Year of publication, Publisher's name and Cost of book. Design a base class containing Accession Number, Name of the author and Title of the book and another base class consisting of year of publication and Publisher's name. Create a derived class having a data member as Cost of the book. Construct member functions to get information on the derived class from the keyboard and display the contents of the class objects on the screen.

UG-C-451 BMS-35

B.Sc. DEGREE EXAMINATION – DECEMBER, 2020.

B.SC., MATHEMATICS

THIRD YEAR

GRAPH THEORY

Time: 3 Hours **Max Marks: 75**

PART – A **(5 X 5 = 25 Marks)**

Answer any five questions.

1. Explain Konigsberg Bridge problem.
2. For a connected graph G, prove that rad G $\leq$ diam G $\leq$ 2 rad G.
3. Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.
4. If G is a graph with p $\geq$3 and $\beta \geq p/2$, prove that G is hamiltonian.
5. In any graph G, prove that a $\beta = p$, with usual notations.
6. Find the chromatic polynomial of a tree.
7. Prove that K_5 is non-planar.
8. Prove that every tournament has a directed Hamilton path.

PART – B **(5 X 10 = 50 Marks)**

Answer any five questions.

1. Prove that the maximum number of edges among all p vertex simple graphs with no triangles in $[p^2/4]$.
2. Prove that a graph is bipartite if and only if it contains no odd cycle.
3. State and prove Haval and Hakimi theorem.
4. State and prove a necessary and sufficient condition for a graph to be eulerian.
5. Define closure of a graph and prove that it is well defined.
6. In any graph G with $^{\delta} > 0$, prove that a1 + β1 = p, with usual notations.
7. State and prove Euler's formula.
8. State and prove five colour theorem.