A132-UG	**BMS–31**

B.Sc. DEGREE EXAMINATION –
SEPTEMBER 2020

MATHEMATICS

THIRD YEAR

REAL AND COMPLEX ANALYSIS

Time : 3 Hours Maximum Marks : 75

PART A — (5 × 5 = 25 Marks)

Answer any FIVE questions.

1. Prove that the set of all rational numbers is countable.

2. Show that the function f defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$.

3. Prove that every closed subset of a compact metric space is closed.

4. If f is bounded and integrable on $[a, b]$, prove that $|f|$ is also bounded and integrable.

5. If $u = x^2 - y^2$ and $v = -\frac{y}{x^2 + y^2}$, prove that $u + iv$ is not an analytic function.

6. Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$.

7. Evaluate $\int_c \frac{z+1}{z(z-1)} dz$ where c is $|z| = 2$.

8. Find the Taylor's series of $f(z) = \frac{z+3}{(z-1)(z-4)}$ valid in $|z-2| < 1$.

PART B — (5 × 10 = 50 Marks)

Answer any FIVE questions.

9. State and prove Minkowski's inequality.

10. Let (X, d) be any metric space. A subset F of X is closed if and only if its complement in X is open.

11. State and prove Lagrange's mean value theorem.

12. Show that continuous image of a connected space is connected.

13. State and prove a sufficient condition for differentiability of complex valued function.

14. Find the bilinear transformation which maps the points $0, 1, \infty$ of the z–plane into the points $i, 1, -i$ of the w–plane respectively.

15. State and prove Taylor's theorem.

16. Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$, using contour integration, where $a > b > 0$.

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B.Sc. DEGREE EXAMINATION —
SEPTEMBER 2020

MATHEMATICS

THIRD YEAR

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 Hours　　　　Maximum Marks : 75

PART A — ($5 \times 5 = 25$ Marks)

Answer any FIVE questions.

1. Show that intersection of two subspaces is also a subspace of a vector space and union of two subspace need not be a subspace.

2. Let $T : U \rightarrow V$ be a linear transformation. Show that range of T is a subspace of V and Kernel of T is a subspace of U.

3. If S and T are subsets of vector space over F, show that $L(SUT) = L(S) + L(T)$.

4. Examine whether the vectors $u = (1, -1, 0)$, $v = (1, 0, -1)$ and $w = (5, 3, -2)$ in R^3 are linearly dependent or linearly independent over R.

5. Show that any vector space V of dimension n over a field F is isomorphic to $V_n(F)$.

6. Let $S = \{v_1, v_2, ...v_n\}$ be an orthogonal set of non – zero vectors in an inner product space V. Then S is linearly independent.

7. Let f be the bilinear form defined on $V_2(R)$ by $f(x, y) = x_1 y_1 + x_2 y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of with respect to the standard basis $\{e_1, e_2\}$

8. Let L be a lattice. Let $a, b \in L$. Show that the following statements are equivalent :

   (a) $a \leq b$

   (b) $a \vee b = b$

   (c) $a \wedge b = a$

PART B — (5 × 10 = 50 Marks)

Answer any FIVE questions.

9. $R^n = \{(x_1, x_2, ... x_n) / x_i \in R, 1 \le i \le n\}$. Show that R^n is a vector space over R under addition and scalar multiplication defined by

$(x_1, x_2, .. x_n) + (y_1, y_2, ... y_n) = (x_1 + y_1, x_2 + y_2, ... x_n + y_n)$ and $a(x_1, x_2, ... x_n) = (a x_1, a x_2, ... a x_n)$.

10. State and prove fundamental theorem of homomorphism.

11. Let A and B be sub spaces of a finite dimensional vector space V. Show that $\dim(A + B) = \dim(A) + \dim(B) - \dim(A \cap B)$

12. Let V be a vector space over a field F and $S = \{v_1, v_2, ... v_n\} \subset V$. Show that the following are equivalent.

(a) S is a basis of V

(b) S is a maximal linearly independent set

(c) S is a minimal generating set.

13. Apply Gram — Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = \{1, 0, 1\}$, $v_2 = \{1, 3, 1\}$ and $v_3 = \{3, 2, 1\}$.

14. Define bilinear form on a vector space V over a field F and show that the set of all bilinear forms on V is a vector space over F.

15. Reduce the quadratic form $x_1^2 + 4x_1 x_2 + 4x_1 x_3 + 4x_2^2 + 16x_2 x_3 + 4x_3^2$ to the diagonal form.

16. Show that the set of all normal subgroups of a group G is a modular lattice.

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B.Sc. DEGREE EXAMINATION — SEPTEMBER 2020

MATHEMATICS

THIRD YEAR

OPTIMIZATION TECHNIQUES

Time : 3 Hours Maximum Marks : 75

PART A — (5 × 5 = 25 Marks)

Answer any FIVE questions.

1. Rewrite the standard form of the following LPP:

   Minimize $z = 2x_1 + x_2 + 4x_3$

   Subject to $-2x_1 + 4x_2 \leq 4$

   $x_1 + 2x_2 + x_3 \geq 5$

   $2x_1 + 3x_3 \leq 2$

   $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

2. Solve by simplex method.

Maximize $Z = 7x_1 + 5x_2$

Subject to $x_1 + 2x_2 \le 6$

$4x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$

3. Obtain the initial basic feasible solution to the transportation problem given below by north west corner rule.

	A	B	C	D	Available
P	19	30	50	10	7
Q	70	30	40	60	9
R	40	80	70	20	18
Demand	5	8	7	14	

4. Give the mathematical formulation of an assignment problem.

5. Solve the game whose pay — off matrix is

	Player B		
Player A	–1	2	–2
	6	4	–6

6. In an inventory model, suppose that the shortages are not allowed and the production rate is infinite and λ = 600units/year, i = 0.20, C_3 = Rs.80.00, p = Rs.3.00 and τ = 1 year. Find the optimum order quantity and reorder point r_k.

7. Write any two characteristics of queueing model.

8. A two person barber shop has 5 chairs to accommodate waiting customers. Potential customers who arrive when all 5 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 4 per hour spend an average of 12 minutes in the barber's chair. Compute P_0.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Using dual simplex method, solve the IPP:

Maximize $Z = -3x_1 - x_2$

Subject to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

10. Solve by using big M method.

Maximize $Z = x_1 + 2x_2 + 3x_3 - x_4$

Subject to
$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_1, x_2, x_3, x_4 \geq 0$$

11. Solve the following transportation problem

	W_1	W_2	W_3	W_4	W_5	Available
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Required	30	30	15	20	5	

12. Solve the following assignment problem:

Job	Machine				
	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

13. Solve the game whose pay — off matrix is

		B I	B II	B III
A	I	–3	–2	6
A	II	2	0	2
A	III	5	–2	–4

14. Solve the following game graphically.

Player A

$$\text{Player B}\begin{pmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & -5 \end{pmatrix}$$

15. Find the optimal order quantity for a product for which the price breaks are as follows:

q	Price per unit (Rs.)
$0 \le q < 500$	10.00
$500 \le q$	9.25

The monthly demand for a product is 200 units. The storage cost is 2% of the unit cost of the product and the cost of ordering is Rs. 350.00/order.

16. A car service station has 2 bays where service can be offered simultaneously. Because of space limitations only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with μ = 8 cars per day.

Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system.

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B.Sc. DEGREE EXAMINATION – SEPTEMBER 2020

MATHEMATICS

THIRD YEAR

PROGRAMMING IN C AND C++

Time : 3 Hours Maximum Marks : 75

SECTION A — ($5 \times 5 = 25$ Marks)

Answer any FIVE of the following.

1. Describe bitwise operators in C.
2. Explain the syntax of switch statement in C.
3. Write short notes on static variables.
4. Write short notes on recursion.
5. How to access a variable through its pointer?
6. Write short notes on arrays of structures.
7. Explain constructors.
8. Write a C program to find the largest number among three numbers.

SECTION B — (5 × 10 = 50 Marks)

Answer any FIVE of the following.

9. Discuss briefly about different data types in C with examples.

10. Explain about different looping statements in C with examples.

11. Explain in detail about Array.

12. Write a C program to access array elements.

13. Explain the concept of opening and closing a file.

14. Explain the following :

    (a) Pointers and character strings

    (b) Array of pointers.

15. Define inheritance. Explain the multiple inheritance with example program.

16. Write a C program to add two matrices using arrays.

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B.Sc. DEGREE EXAMINATION — SEPTEMBER 2020

MATHEMATICS

THIRD YEAR

GRAPH THEORY

Time : 3 Hours | Maximum Marks : 75

PART A — (5 × 5 = 25 Marks)

Answer any FIVE questions.

1. Draw a graph with 5 vertices, a, b, c, d, e such that deg $(a) = 3, b$ is an odd vertex. $\deg(c) = 2$, d and e are adjacent.

2. If a graph has n vertices and a vertex v is connected to a vertex w then prove that there exists a path from v to w of length no more than $n-1$.

3. Define a bipartite graph and give an example.

4. Prove that in a tree any two vertices are connected by a unique path.

5. Let G be a simple graph and let u and v be nonadjacent vertices in G such that $d(u)+d(v) \geq v$. Prove that G is Hamiltonian if and only if $G+uv$ is Hamiltonian.

6. If G is k-critical then prove that $\delta \geq k-1$.

7. Prove that a graph G is embeddable in the plane if and only if it is embeddable on the sphere.

8. Define $d^{-}(v)$ and $d^{+}(v)$ in a digraph D. Also prove that $\sum d^{-}(v) = \sum d^{+}(v)$.

PART B — (5 × 10 = 50 Marks)

Answer any FIVE questions.

9. Prove that in any graph, the number of vertices of odd degree is even.

10. Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

11. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

12. Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

13. Prove that a nonempty connected graph is eulerian if and only if it has no vertices of odd degree.

14. If G is simple then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.

15. Prove that K_5 is nonplanar.

16. Prove that a digraph D contains a directed path of length $\chi - 1$.