TAMIL NADU OPEN UNIVERSITY

Chennai - 15

School of Science

ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-15, Algebra
Batch : CY 2020
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. If V is finite dimensional inner product space and W a subspace of V then prove that V is the direct sum of W and $W^{\perp}$.
2. Let f(x) ∈ *F*[x] be of degree n≥1. Then prove that there is an extension E of F of degree at most n! in which f(x) has n roots.
3. State and prove fundamental theorem of Algebra.
4. Prove that every finite abelian group is the direct product of its sylow subgroups.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. If $p(x)$ is irreducible in $F[x]$ and if V is a root of $p(x)$ then prove that $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(t)$. Also prove that this isomorphism σ can so be chosen such that $\sigma: F(v) \to F'(v)$ 1. $v\sigma = w$ 2. $\alpha\sigma = \alpha'$ for every $\alpha \in F$.
2. Prove that S_n is not solvable for n ≥ 5.
3. Prove that $F^{(n)}$ is isomorphic to $F^{(m)}$ iff n = m.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Show that every abelian group is a module over the ring of integers.
2. Find the splitting filed of x^4 ï $5x^2 + 6 \in Q(x)$.
3. State and Prove Cayley's Theorem.
4. State and prove Fermat Theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. If a and b in K are algebraic over F of degrees m and n respectively, then prove that $a \pm b$, ab and a/b (if b$\neq$ 0) are algebraic over F of degree at most mn.

2. Prove that any splitting Fields E and E' of the polynomials $f(x) \in F[x]$ and $f'(t) \in F'[t]$ respectively are isomorphic by an isomorphism ϕ with the property that $\alpha\phi = \alpha$ for every $\alpha \in F$. (In particular, any two splitting fields of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of F fixed).

3. If V is finite dimensional and W is subspace of V, then prove that $\hat{W}$ is isomorphic to $\hat{V}$/A(W) and dim A(W) = dim V ï dim W.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. If L is a finite extension of a field F and K be a sub-field of L containing F, then prove that

   [K:F]/[L:F].

2. Find the splitting field of the polynomial $x^4 + 1 \in Q(x)$.

3. If G and G' are isomorphic abelian groups, then for every integer s, prove that G(s) and G'(s) are isomorphic, with usual notation.

4. The ideal I = (p(*x*)) in F[*x*] is a maximal ideal iff p(*x*) is irreducible over F.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Any finite extension of a field of characteristic zero is a simple extension.
2. Write a note on Nilpotent Transformations.
3. Write a note on Construction with straight edge and compass.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that a polynomial of degree n over a field can have at most n roots in any extension field.
2. Show that it is impossible, by straight edge and compass alone, to trisect 60^{o}.
3. Let G be an abelian group and a, b ∈ G be of order m and n respectively. Show that there exists an element c of G such that o(c) = k where k is LCM of m and n.
4. Let V be a finite dimensional vector space over the field F and W be a subspace of V. Then prove that A(A(W)) = W.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that Homomorphic image of a solvable group is solvable.
2. Prove that the invariants of a nilpotent transformations T is unique.
3. If p(x) is a polynomial in F[*x*] of degree n≥1 and is irreducible over F, then prove that there is an extension E of F such that [E:F] = n, in which p(*x*) has a root.

TAMIL NADU OPEN UNIVERSITY

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School of Science

ASSIGNMENT

Programme Code No	: 231
Programme Name	: M.Sc., Mathematics
Course Code & Name	: MMS-16, Real Analysis
Batch	: CY 2020
No.of Assignment	: One Assignment for Each 2 Credits
Maximum Marks	: 100
Weightage	: 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that continuous functions are integrable.
2. State and prove Abel's Theorem.
3. Prove that any infinite subset of a compact set has a limit point.
4. State and prove Merten's Theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that Uniform limit of integrable functions is integrable.
2. State and prove Rank Theorem.
3. Write a note on "Continuity and Compactness".

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove fundamental theorem of calculus.
2. State and prove Taylorõs Theorem.
3. Show that any non-empty perfect set in R^k is uncountable.
4. Show that convex functions are continuous.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that there exists real continuous function on the real line which is nowhere differentiable.
2. Write a note on exponential functions and logarithmic functions.
3. Write a note on ñContinuity and Connectednessò

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that C(X) is a complete metric space under the metric induced by the supremum norm.
2. Show that the function E is periodic with period $2\pi i$.
3. Prove that every interval [a,b], a < b, is uncountable.
4. State and prove LôHospitals rule.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Prove that if f is a continuous complex function on [a, b], there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x)$ = f(x) uniformly on [a, b].
2. State and prove inverse functions theorem.
3. Write a note on mean value theorems.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Bessel's inequality.
2. State and prove Strings Formula.
3. Prove that e is irrational.
4. Does L' Hospitals rule hold for vector valued functions? Justify your answer.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Implicit function theorem.
2. Discuss whether continuity, differentiability, integrability and limit process are preserved under limit operations.
3. Write a note on Riemann-Stieltjes Integration.

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School of Science

ASSIGNMENT

Programme Code No	: 231
Programme Name	: M.Sc., Mathematics
Course Code & Name	: MMS-17, Complex Analysis and Numerical Analysis
Batch	: CY 2020
No.of Assignment	: One Assignment for Each 2 Credits
Maximum Marks	: 100
Weightage	: 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Bisection method and illustrate.
2. Write a note on divided differences.
3. Explain Stereographic projection.
4. State and prove Taylor's Theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Solve the following equation using Guass-Jacobi iteration method.

   $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$.

2. Solve : $\frac{dy}{dx} = x + y$, $y(0) = 1$ by the Picard method of successive approximations.

3. Derive Simpson's one –third rule and Apply Simpson's one –third rule to evaluate the approximate value of the integral by dividing the range into 8 equal parts.

   $$\int_{2}^{10} \frac{dx}{1+x}$$

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Find a real root of the equation $x^3 - 2x - 5 = 0$ by secant method.
2. Explain Hermite interpolations method.
3. Prove that every analytic function is independent of $\bar{z}$.
4. State and prove the maximum principle.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Gauss - jordan method and solve the following system of equations by Gauss jordan method.

   $10x_1 + x_2 + x_3 = 12, \quad x_1 + 10x_2 + x_3 = 12, \quad x_1 + x_2 + 10x_3 = 12.$

2. Use power method to find the dominant eigen value and eigenvector of

   $$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

3. Explain Milne method and apply to find y(1.0) given that $y' = x - y^2$, $y(0) = 0$.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Cholesky method.
2. Show that the value of the divided difference does not depend upon the order of the arguments involved in it.
3. State and prove Abel limit theorem.
4. Explain Laurent series development and analytic function.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Gauss elimination method and solve the following system of equations by Gauss elimination method.

   $10x_1 + x_2 + x_3 = 12$, $x_1 + 10x_2 + x_3 = 12$, $x_1 + x_2 + 10x_3 = 12$.

2. Solve the system of equations by Gauss-Seidel method.

   $8x - y + z = 18$, $2x + 5y - 2z = 3$, $x + y - 3z = -6$.

3. Using modified Euler method, find the value of y when x = 0.3, given that $\frac{dy}{dx} = x + y$, y(0)=1.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and explain Lagrange�s formula on interpolation.
2. By Taylor series method, find the value of y at x = 0.1 and x = 0.2 from $\frac{dy}{dx} = x^2y - 1$,

   y(0) = 1.
3. State and prove integral formula.
4. State and Argument principle.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Use Bisection method to find the approximate value of the root of the equation $3x - \sqrt{1 + \sin x}$

   = 0.
2. Expand $\frac{z}{(z-1)(2-z)}$ in Laurent series valid for i) $|z| < 1$, ii) $1 < |z| < 2$, iii) $|z| > 2$ iv) $|z - 1| > 1$,

   (v) $0 < |z - 2| < 1$.
3. Solve : y' = xy, y(1) = 2, for x = 1.4 using Runge-Kutta method.

TAMIL NADU OPEN UNIVERSITY

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-18, Mathematical Statistics
Batch : CY 2020
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Given that probability density function $f\,(x;\theta) = \frac{1}{\pi[1+(x-\theta)^2]}$, $-\infty < x, -\infty$, $-\infty < \theta, -\infty$. Show that the Rao – Cramer' lower bound is $\frac{2}{n}$, where n is the size of a random sample from this Cauchy distribution.

2. Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size n from a distribution that is n (μ, σ^2). Then the random variable $Y = \sum_{i=1}^{n}\left(\frac{X_i-\mu}{\sigma}\right)^2$ has a χ^2 distribution with n degrees of freedom.

3. Define negative binomial distribution and find the moment generating function of a negative binomial distribution.

4. State and prove Rao – Blackwell theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size $n \geq 2$ from a distribution that is n (μ, σ^2). Let $\overline{X}$ and S^2 be the mean and variance of this random sample. Then

   (a) $\overline{X} \sim n\left(\mu, \frac{\sigma^2}{n}\right)$
   (b) $n\frac{S^2}{\sigma^2} \sim \chi^2\,(n-1)$ and
   (c) $\overline{X}$ and S^2 are stochastically independent.

2. Write a note on Binomial Distribution .

3. Write a note on Stochastic convergence.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Let X_1, X_2, …, X_n denote a random sample from a Poisson distribution, that has the mean $\theta > 0$. Prove that $\bar{X}$ is an efficient estimator of θ.

2. Let X_1, X_2, X_3, …, X_n be mutually stochastically independent random variables that have, respectively, the Chi – square distributions of r_1, r_2, r_3, … , r_n degrees of freedom. Then the random variable $Y = X_1 + X_2 + X_3 + \ldots + X_n$ has a chi square distribution with $r_1 + r_2 + r_3 + \ldots + r_n$ degrees of freedom.

3. Define trinomial distribution and find the moment generating function of trinomial distribution.

4. State and prove Neyman – Pearson theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let X_1, X_2, X_3 denote a random sample of size 3 from a distribution that is n(0,1). Find the probability density function of $Y = X_1^2 + X_2^2 + X_3^2$.

2. Write a note on Poisson Distribution.

3. Show that $Y = \frac{1}{1+\frac{r_1}{r_2}F}$ where F has an F distribution with parameters r_1 and r_2 has a beta distribution.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Sequentially Probability Ratio Test.
2. Let F have an F – distribution with parameters r_1 and r_2 then $\frac{1}{F}$ has an F distribution with parameters r_2 and r_1.
3. Find the moment generating function of Gamma distribution and also find its mean and variance.
4. Let $\bar{X}$ denote the mean of a random sample of size 128 from a gamma distribution with α=2 and β = 4. Find Pr(7 < $\bar{X}$ < 9).

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let the random variable X have the probability density function f (x) = $\begin{cases} 1\,; 0 < x < 1 \\ 0\,; \text{elsewhere} \end{cases}$

   Let X_1 and X_2 denote a random sample from this distribution. Find the joint probability density function of Y_1 and Y_2 where $Y_1 = X_1 + X_2$, $Y_2 = X_1 - X_2$. Find also marginal probability density function of Y_1 and Y_2.

2. Let f (x,y) = 2, 0 < x < y, 0 < y < 1, zero elsewhere, be the joint probability density function of X and Y. show that the conditional means are, respectively $\frac{1+x}{2}$, 0 < x < 1 and $\frac{y}{2}$, 0 < y < 1. show also that the correlation coefficient of X and Y is $\rho = \frac{1}{2}$.
3. Find the moment generating function of a bivariate normal distributions.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Central limit theorem.

2 . State and prove Rao – Cramer inequality.

3. Bowl I contains 3 red chips and 7 blue chips. Bowl II contains 6 red chips and 4 blue chips. A bowl is selected at random and then 1 chip is drawn from this bowl.
   (i) Compute the probability that this chip is red.
   (ii) Relative to the hypothesis that the chip is red, find the conditional probability that it is drawn from bowl II.

4. State and prove Chebyshev's inequality.

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Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let Z_n be χ^2 (n). Find the limiting distribution of the random variable $y_n = \frac{z_n - n}{\sqrt{2n}}$.

2. Write a note on Conditional Probability.

3. Write a note on Normal Distribution.