TAMIL NADU OPEN UNIVERSITY

Chennai - 15

School of Science

ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-25, Topology and Functional Analysis
Batch : CY 2020
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that the topologies induced by the Euclidean metric and the square metric are the same as the product topology on R^n
2. Write a note on quotient topology.
3. Define a Hausdorff space, give an example and prove that every finite set in a Hausdorff space is closed.
4. State and prove Uniform limit theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and Prove Tietze extension theorem.
2. State and Prove Uryshon's metrization theorem.
3. Prove that every Hilbert space is reflexive.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove the maximum and minimum value theorem.
2. Prove that If X is limit point compact then it is sequentially compact.
3. Define a connected space, give an example and prove that the continuous image of a connected space is connected.
4. Define a compact space, give an example and a counter example

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and Prove Hahn-Banach Theorem, proving necessary result.
2. Prove that If N is normed linear space, then the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.
3. State and prove Open Mapping theorem.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Uniform Boundedness theorem.
2. Write a note on the natural imbedding of N in N^{**}.
3. Define a Regular space and prove that product of Regular space is Regular.
4. Prove that every metrizable space is normal.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Closed Graph theorem, proving necessary result.
2. State and prove Bessel's inequality.
3. If $\{e_i\}$ is an orthonormal set in a Hilbert space H and x is an arbitrary vector in H, then prove that

   $[x - \sum(x, e_i)e_i] \perp e_j$ for each j.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Gram-Schmidt orthogonalisation process.
2. List and prove properties of adjoint operator on a Hilbert space.
3. State Polarisation of identity and prove.
4. Prove that the inner product in a Hilbert space is jointly continuous.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Let H be a Hilbert space and let $\{e_i\}$ be an orthonormal set in H. Then the following conditions

   are equivalent to one another
   (i) $\{e_i\}$ is complete.
   (ii) $x \perp e_i \ \forall i$ implies x = 0.
   (iii) If x is an arbitrary vector in H, then $x = \sum(x, e_i)e_i$.
   (iv) If x is an arbitrary vector in H, then $||x||^2 = \sum|(x, e_i)|^2$.

2. State and prove Riesz Representation Theorem.
3. If T is a positive operator on a Hilbert space H, then prove that I + T is non-singular.

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ASSIGNMENT

Programme Code No	: 231
Programme Name	: M.Sc., Mathematics
Course Code & Name	: MMS-26, Operation Research
Batch	: CY 2020
No.of Assignment	: One Assignment for Each 2 Credits
Maximum Marks	: 100
Weightage	: 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Two-Phase Technique.
2. Explain Dual simplex Algorithm.
3. Explain production allocation problem and convert it in to a LPP.
4. Find the Dual of the following LPP

   Max $Z = 3x_1 + x_2 + 2x_3 - x_4$ subject to the constraints

   $2x_1 - x_2 + 3x_3 + x_4 = 1$, $x_1 + x_2 - x_3 + x_4 = 3$, $x_1, x_2 \geq 0$, x_3 and x_4 are unrestricted.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Use Simplex method to solve the following L.P.P.
   Maximize $Z = 7x_1 + 5x_2$ subject to the constraints
   $x_1 + 2x_2 \leq 6$, $4x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$

2. Use two-phase simplex method to minimize $Z = \frac{15}{2}x_1 - 3x_2$
   Subject to the constraints $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.

3. Use dual simplex method to solve the L.P.P.

   Minimize $Z = x_1 + 2x_2 + 3x_3$ subject to the constraints
   $x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$, $x_2 - x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain the Augmenting Path Algorithm for the maximum flow problem.
2. Explain Probabilistic dynamic programming.
3. Write a note on network analysis.
4. Explain the minimum spanning tree problem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Illustrate interior – point algorithm.
2. Apply minimum Spanning Tree algorithm to obtain solution to the Seervada Park problem.
3. Explain the Augmenting Path Algorithm to the Seervada Park problem.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain all I.P.P. Algorithm.
2. Explain Branch-and-Bound Algorithm.
3. Explain two person ï zero sum game illustrate.
4. Solve the following 5 x 2 game graphically

	B_1	B_2
A_1	-2	5
A_2	-5	3
A_3	0	-2
A_4	-3	0
A_5	1	-4

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Solve the following 3 x 3 game by linear programming

Player B

Player A $\begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}$

2. Find the optimum integer solution to the following all I.P.P.

Maximize $Z = x_1 + 2x_2$
Subject to the constraints
$x_1 + x_2 \leq 7$, $2x_1 \leq 11$, $2x_2 \leq 7$, $x_1, x_2 \geq 0$ and are integers.

3. Use Branch-and Bound technique to solve the following I.P.P.
Maximize $Z = 7x_1 + 9x_2$
Subject to the constraints
$-x_1 + 3x_2 \leq 6$, $7x_1 + x_2 \leq 35$, $0 \leq x_1, x_2 \leq 7$, x_1, x_2 are integers.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Explain Geometric Programming.
2. Write a note on Quadratic programming.
3. Explain the Direct search method in unconstrained non linear algorithms.
4. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter – arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes, calculate the following:
   i) Average number of trains in the yard.
   ii) The probability that the queue size exceeds 10.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Pure Birth and Death Model.
2. Explain Specialized Poisson Queues – M/A/1 Queue.
3. Write a note on Separable Convex Programming.

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ASSIGNMENT

Programme Code No : 231
Programme Name : M.Sc., Mathematics
Course Code & Name : MMS-27, Graph Theory and Algorithms
Batch : CY 2020
No.of Assignment : One Assignment for Each 2 Credits
Maximum Marks : 100
Weightage : 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Write a note on Degree Sets.
2. Estimate the Maximum number of edges among all p vertex simple graphs with no triangles.
3. Explain Breadth First search Algorithm.
4. State and prove Cayley's Formula.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

3. State equivalent conditions for a graph to be a Tree and Prove.
1. State and prove Menger's Theorem.
2. Write a note on "Operations of Graphs".

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Prove that an eulerian graph G is randomly eulerian from a vertex v if and only if every cycle of G contains v.
2. If G is a graph with p $\geq$ 3 and $\delta \geq$ p/2, then prove that G is hamiltoniam.
3. State and prove a necessary and sufficient condition for a graph to be 2-connected.
4. State and prove Haval and Hakimi Theorem.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Define an Eulerian Graph, give example and a counter example. Also state and prove a necessary and sufficient condition for a connected graph to be Eulerian.
2. Explain Marriage Problem, proving Hall's Marriage Theorem.
3. State and prove Tutte's Theorem.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Write a note on Uniquely colourable Graphs.
2. Define a critical graph and prove that every critical graph is a block.
3. State and prove a necessary and sufficient condition for a matching to be maximum matching.
4. In any graph G with $\delta > 0$, prove that $\alpha' + \beta' = p$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain Stable Matching and prove that for every assignment of preferences in a bipartite graph, there is a stable matching.
2. State and prove Vizing's Theorem.
3. State and prove Brooks Theorem.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Write a note on Geometric Dual.
2. State and prove Five-colour theorem.
3. Define Chromatic polynomial and prove that the chromatic polynomial of tree of order p

   is $k(k-1)^{P-1.}$
4. State and prove Euler's Formula.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Write a note on Mycielski's Construction.
2. Define the chromatic polynomial of a graph and list some properties of chromatic polynomial and justify your answer.
3. Prove the following are equivalent:

   a) Every planner graph is 4-vertex colourable.

   b) Every plane graph is 4-face colourable.

   c) Every simple 2-edge connected 3-regular planar graph is 3-edge colourable.

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ASSIGNMENT

Programme Code No	: 231
Programme Name	: M.Sc., Mathematics
Course Code & Name	: MMS-28, Differential Equations
Batch	: CY 2020
No.of Assignment	: One Assignment for Each 2 Credits
Maximum Marks	: 100
Weightage	: 25%

Assignment – I

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Solve : $3y' + y = 2e^{-x}$
2. Solve : $2y'' – 5y' + 3y = 0$.
3. Explain variation of parameter to find a particular solution of non-homogenous equations.
4. Write a note on Wronskian.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove the Existence Theorem for the initial value problem of a second order linear homogeneous differential equation.
2. Let $\emptyset_1, \emptyset_2, \ldots, \emptyset_n$ are n solutions of a linear differential equation L(y) = 0 on an interval I. Then prove that $\emptyset_1, \emptyset_2, \ldots, \emptyset_n$ are independent on I if and only if $W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x) \neq 0$ for all x in I.
3. Let $\{\emptyset_1, \emptyset_2, \ldots, \emptyset_n\}$ be *n* solutions of a linear homogeneous equation with constant coefficients L(y) = 0 on an interval I containing a point x_0. Then prove that $W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x) = e^{-a1(x-x0)} W(\emptyset_1, \emptyset_2, \ldots, \emptyset_n)(x_0)$.

Assignment – II

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Solve : $y'' + y = 0$, $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 2$.

2. Show that, for any n and m with $n \neq m$,

   $\int_{-1}^{1} P_n(x)\, P_m(x) = 0.$

3. Show that, for any n, $P_n(-x) = (-1)^n P_n(x)$ and hence $P_n(-1) = (-1)^n$, with usual rotation.

4. Solve : y'' ï 2y' ï 3y = 0, y(0) = 0, y'(0) = 4.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Explain the Algorithm of variation of Parameter method and also find a particular solution of $y'' + y = \operatorname{cosec} x$.

2. Find the solution of the initial value problem
   $y'' - 2y' + y = 2x$, $y(0) = 6$, $y'(0) = 2$.

3. Prove that for each n there is one and only one polynomial solution $P_n(x)$ of degree n for the Legendre equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0$ satisfying $P_n(1) = 1$.

Assignment – III

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. Find a solution of $(D^2 - D')z = 2y - x^2$.

2. Find a particular integral of $(D^2 - D')z = e^{x+y}$.

3. Find the general solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$

4. Solve $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{\partial^4 z}{\partial x^2 \partial y^2}$.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. Define Bessel Equation and solve.
2. Find a fundamental matrix of the equation

$$y' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix} y$$

3. Solve the initial value problem y'' – 2y' + y = 0, y(0) = 0, y'(0) = 1 on the interval [0,a] where a is any positive real number.

Assignment – IV

Part – A (4 x 10 = 40 Marks)

Answer all questions. Each question carries 10 marks.

1. State and prove Kelvin's inversion theorem.
2. Write a note on Neumann's problem.
4. Find a solution for the Laplace's equation.
5. State and prove a necessary condition for a general one-parameter family of surfaces to be a family of equipotential surfaces.

Part – B (2 x 30 = 60 Marks)

Answer any two of the questions. Each question carries 30 marks.

1. State and prove Picard's Theorem.
2. Reduce the equation into its canonical form: $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$.
3. Write a note on Characteristic Curves.