PG – 769	PGDAM–11

P.G. DIPLOMA IN APPLIED MATHEMATICS EXAMINATION — DECEMBER, 2019.

(AY 2003–04 and CY–2004 batch only)

OPERATIONS RESEARCH

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

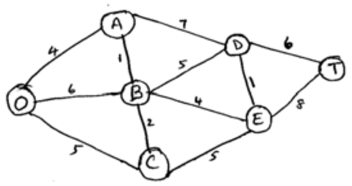
Answer any FIVE questions

1. What are the major assumptions and limitations of linear programming.

2. A firm produces two products say X and Y. Product X sells for a net profit of Rs. 80 per unit, while product Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a goal peogramming model.

3. What is dynamic programming?

4. Consider the following networks where the numbers on links represent actual distances between the corresponding nodes. Find the minimal spanning tree:



5. Solve the following game:

Player B

		I	II	III	IV
Player A	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

6. Solve the following game by graphical method

Player B

Player A $\begin{pmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{pmatrix}$

7. A supermarket has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour.

Calculate:

(a) the probability that the cashier is idle.

(b) the average number of customer in the queuing system.

(c) the average time a customer spends in the system.

(d) the average number of customer in the queue.

(e) the average time a customer spends in the queue waiting for services.

8. Use separable convex programming to solve the non linear programming problem:

Maximize $f(x) = 3x_1 + 2x_2$

Subject to the constraints:

$g(x) = 4x_1^3 + x_2^2 \leq 16$ and $x_1, x_2 \geq 0$.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Use simplex method to solve the following L.P.P.

Maximize $z = 4x_1 + 10x_2$

Subject to the constraints:

$2x_1 + x_2 \leq 50,$
$2x_1 + 5x_2 \leq 100,$
$2x_1 + 3x_2 \leq 90,$
$x_1 \geq 0$ and $x_2 \geq 0.$

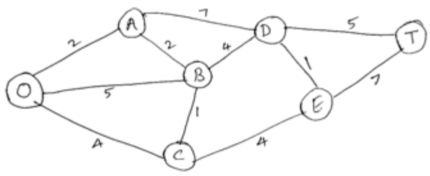
10. Use dual simplex method to solve the LPP

Maximize $z = -3x_1 - 2x_2$

Subject to the constraints:

$x_1 + x_2 \geq 1,$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10,$
$x_2 \leq 3$ and
$x_1, x_2 \geq 0.$

11. Determine a shortest path from O to T for the following network:



12. The following table lists the jobs of a network along with their estimates:

Job		Duration (days		
i	j	Optimistic	Most likely	Pessimistic
1	2	3	6	15
1	6	2	5	14
2	3	6	12	30
2	4	2	5	8
3	5	5	11	17
4	5	3	6	15
6	7	3	9	27
5	8	1	4	7
7	8	4	19	28

(a) Draw the project network.

(b) Calculate the length and variance of the critical path.

(c) What is the approximate path will be completed in 45 days?

(d) What is the chance of project duration exceeding 46 days?

13. Solve the following game:

		Player B			
		I	II	III	IV
	1	6	8	3	13
Player A	2	4	1	5	3
	3	8	10	4	12
	4	3	6	7	12

14. Solve the following integer programming problem using the cutting-plane algorithm:

Maximize $Z = 3x_1 + x_2 + 3x_3$

Subject to the constraints:

$-x_1 + 2x_2 + x_3 \leq 4,$
$4x_2 - 3x_3 \leq 2$
$x_1 - 3x_2 + 2x_3 \leq 3,$
x_1, x_2 and x_3

all are non-negative integers.

15. Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. the service time for each train is assumed to be exponential with an average of 36 minutes. if the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purposes) Calculate the probability that the yard in empty and find the average queue length.

16. Using geometric programming, solve the following problem:

    Minimize $f(x) = 5x_1^{-1}x_2^{-1}x_3^{-1} + 5x_2x_3$

    Subject to the constraints:

    $2x_1x_3 + x_1x_2 = 4,\ x_1, x_2, x_3 > 0.$

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PG – 770	PGDAM–12

PG DIPLOMA IN APPLIED MATHEMATICS EXAMINATION — DECEMBER, 2019.

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions

1. Prove that, in any graph G the number of points of odd degree is even.

2. Prove that every connected graph has a spanning tree.

3. Prove that for any graph G, $k \le \lambda \le \delta$ where $k = k(G)$ is the minimum number of points whose removal results in a disconnected graph and $\lambda = \lambda(G)$ is the minimum number of lines whose removal results in a disconnected graph.

4. Show that a partition $P = (d_1, d_2 ..., d_p)$ of an even number into p parts with $p-1 \geq d_1 \geq d_2 \geq ... \geq d_p$ is graphical iff the modified partition $\rho' = (d_2 - 1, d_3 - 1*, ..., d_{d+1} - 1, d_{d+2}{}^{..., d_p})$ is graphical.

5. Prove that c(G) is well defined.

6. Find the number of perfect matchings in the complete graph K_{2n}.

7. Prove that every uniquely n-colourable graph is (n-1) connected.

8. Prove that a graph is planar if and only if it has no subgroup homeomorphic to K_5 or $K_{3,3.}$

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions

9. Explain Breadth-First search algorithm with an example.

10. Prove that there are n^{n-2} distinct labelled trees on n vertices.

11. State and prove Merger's theorem.

12. Prove that, a group G with atleast two points is bipartite iff all its cycles are of even length.

13. If G is a simple graph with $n(\geq 3)$ vertices, and if $\deg(v) + \deg(w) \geq n$ for each pair of non-adjacent vertices v and w, then show that G is Hamiltonian.

14. Prove that a necessary and sufficient conditions for a solution of the marriage problem is that each set of k girls collectively known as atleast k boys for $1 \leq k \leq m$.

15. Explain sequential colouring Algorithm with an example.

16. If G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively, then show that $|V| - |E| + |F| = 2$.

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PG–771 **PGDAM–13**

PG DIPLOMA EXAMINATION —
DECEMBER 2019.

Applied Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. If $A_1, A_2, A_{3,\ldots\ldots,} A_n$ are the n events, then prove that

(a) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

(b) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{u=1}^{n} P(A_i)$

2. The contents of urns I, II and III are as follows:

(a) 1 white, 2 black and 3 red balls,

(b) 2 white, 1 black and 1 red balls and

(c) 4 white, 5 black and 3 red balls.

One urn is chosen at random and two balls drawn from it. They happen to be white and red. What is the probability that they come from urns I, II or III?

3. Obtain the joint p.d.f. of two order statistics.

4. Show that for a random sample of size two from $N(0,\sigma^2)$ population, $E[X_{(1)}] = -\frac{\sigma}{\sqrt{\pi}}$.

5. Define the invariance property of a consistent estimator and establish it.

6. Obtain $100(1-\alpha)\%$ confidence intervals for the parameters

   (a) θ and

   (b) σ^2, of the normal distribution.

7. Construct Likelihood Ratio test for testing $H_o : \theta = \theta_0$ against all its alternatives in $N(\theta, \sigma^2)$ where σ^2 is known.

8. Let $X_1, X_2, ..., X_n$ be a random sample from a distribution with p.d.f. :

   $f(x,\theta) = e^{-x^{(x-\theta)}}, 0 < x < \infty, -\infty < \theta < \infty$ obtain sufficient statistic for θ.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. The joint p.d.f. of x and y is

$$f(x,y)=\begin{cases}k(4-x-y), & 0\le x,y\le 2\\ 0, & \text{otherwise}\end{cases}$$

Find

(a) the constant k

(b) $E(X)$ and $E(Y)$

(c) $E(x/y)$

(d) Var (X) and Var (Y)

10. Derive the $f-$distribution on (v_1,v_2) degress of freedom.

11. State and prove Lindberg-Levy central limit theorem:

12. State and prove khinchin's theorem for the law of large number.

13. State and prove Cramer-Rao inequality.

14. Let $\{T_n\}$ be a sequence of estimators such that for all $\theta\in\Theta$,

(a) $E_\theta(T_n)\to\gamma(\theta), n\to\infty$ and

(b) $Var_\theta(T_n)\to 0$ as $n\to\infty$. Then prove that T_n is a consistent estimator of $\gamma(\theta)$.

15. State and prove Neyman-Pearson Lemma.

16. Explain the test for the mean of a normal population in Likelihood Ratio Test.
