PG – 704 **MMS–15/ PGDMAT – 11**

M.Sc.DEGREE / P.G DIPLOMA EXAMINATION —DECEMBER, 2019.

First Year

Mathematics

ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. State and prove Cauchy's theorem for abelian groups.

2. Let G be a finite group and suppose that G is a subgroup of the finite group M. suppose further that M has a p-Sylow subgroup Q. Then prove that G has a p-Sylow subgroup P.

3. If R is a ring, then for all a, $b \in R$. Prove that

   (a) $a0 = 0a = 0$

   (b) $a(-b) = (-a)b = -(ab)$

   (c) $(-a)(-b) = ab$.

   in addition, R has a unit element 1, then

   (d) $(-1)a = -a$

   (e) $(-1)(-1) = 1$

4. If V is finite-dimensional and if W is a subspace of V, then show that W is finite-dimensional, dim W ≤ dim V and dim V/ W = dim V - dim W.

5. If $u,v \in V$ then prove that $\|(u,v)\| \leq \|u\|\|v\|$.

6. Prove that “The polynomial f(x) ∈F[x] has a multiple root if and only if f(x) and f'(x) have a nontrivial common factor”.

7. If *p(x)* ∈ *F[x]* is solvable by radicals over F, then prove that the Galois group over *F* of *p(x)* is a solvable group.

8. If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K. Then prove that $G/K \approx \overline{G}$.

10. Let G be an abelian group of order p^n, p a prime. Suppose $G = A_1 \times A_2 \times .... \times A_k$, where each Ai = ($a_i$) is cyclic of order p^{n1} and $n_1 \geq n_2 \geq .... \geq n_k > 0$. If m is an integer such that $n_t > m \geq n_{t+1}$ then G(p^m) = $B_1 \times .... \times B_t \times A_{t+1} \times ... \times A_k$ where B_i is cyclic of order p^m, generated by $a_i^{p^{n_i - m}}$ for i ≤ t. Prove that the order of G (p^m) is p^u, where $u = mt + \sum_{i=t+1}^{k} n_i$.

11. State and prove unique factorization theorem

12. If V and Ware of dimensions m and n, respectively, over F, then prove that Hom *(V,W)* is of dimension *mn* over *F*.

13. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, [L:F] = [L:K][K:F].

14. Prove that the number e is transcendental.

15. If K is a finite extension of F, then prove that G(K, F) is a finite group and its order, o(G(K, F)) satisfies o(G(K, F)) $\leq$ [K:F].

16. If T$\in$A(V) has all its characteristic' roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

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PG–705	**MMS–16/ PGDMAT–12**

M.Sc. DEGREE / P.G. DIPLOMA EXAMINATION — DECEMBER, 2019.

First Year

Mathematics

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that infinite subset of a countable set is countable.

2. State and prove the ratio test .

3. Prove that A mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

4. Suppose f is continuous on $[a,b]$, $f(x)$ exists at some point $x \in [a,b]$, g is defined on an interval I

which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t))(a \leq t \leq b)$, then h is differentiable at x, and $h'(x) = g'(f(x))f(x)$.

5. Suppose f is a continuous mapping of $[a, b]$ into R^k and f is differentiable in (a, b). Then there exists $x \in (a, b)$ such that $\left| f(b) - f(a) \right| \leq (b-a)\left| f(x) \right|$.

6. State and prove the L-Hospital rule.

7. State and prove Fundamental theorem of calculus.

8. State and prove inverse function theorem.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Show that countable union of countable sets is countable.

10. State and prove the root test.

11. Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

12. Prove that $f \in R[a, b]$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

13. Suppose $f \in R[a, b]$ on $[a, b]$ $m \le f \le M$, Φ is continuous on $[m, M]$, and $h(x) = \Phi\ (f(x))$ on $[a, b]$. Then prove that $h \in R[a, b]$.

14. Prove that The sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\in > 0$ there exists an integer N such that $m \ge N$, $n \ge N$, $x \in E$ implies $|f_n(x) - f_m(x)| \le \in$.

15. State and prove Stone-Weierstrass theorem.

16. State and prove Implicit function theorem.

PG – 706	**MMS–17**

M.Sc. DEGREE EXAMINATION –
DECEMBER, 2019.

First Year

Mathematics

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours   Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that an analytic function with constant modulus is constant.

2. Find a bilinear transformation which maps $z = 1, 0, -1$ of z-plane into $w = i, 0, -i$ of w-plane.

3. State and prove Liouville's theorem.

4. Solve the following system of equations by Gauss-Jordan method.

   $x + 2y + z = 3$

   $2x + 3y + 3z = 10$

   $3x - y + 2z = 13$

5. Find a positive root of $xe^x = 2$ by the method of false position.

6. Find the value of y at $x = 21$ from the following data.

x :	20	23	26	29
y :	0.3420	0.3907	0.4384	0.4848

7. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Trapezoidal rule with $h = 0.2$.

8. Using Euler's method, solve the equation $y' = x + y,\ y(0) = 1$ for $x = 0.2, 0.4$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove the sufficient condition for $f(z) = u + iv$ to be analytic is a domain D.

10. State and prove Laurent's expansion theorem.

11. Show that $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}$.

12. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 4 decimal places.

13. Solve the following system of equations by Gauss Seidal method

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

14. Derive Lagrange's formula for interpolation.

15. Using R.K. method of fourth order, find $y(0.8)$ correct to 4 decimal places if $y' = y - x^2$, $y(0.6) = 1.7379$.

16. Using Adam's method, find $y(0.4)$ given

$\frac{dy}{dx} = \frac{1}{2}xy$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$.

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PG–707	**MMS–18**

M.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Define continuous distribution function and State its properties.

2. State and Prove the addition theorem of expectation.

3. If the events A and B are such that $P(A) \neq 0, P(B) \neq 0$ and A is independent of B, then prove that B is independent of A.

4. Let $x_1, x_2, \ldots, x_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, \ldots, X_n)$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ.

5. If $x_1, x_2, \ldots x_n$ is a random sample from a normal population $N(\mu, 1)$. Show that an $t = \frac{1}{n}\sum_{i=1}^{n} x_i^2$, unbiased estimator of $\mu^2 + 1$

6. If T is an unbiased estimator for a θ, show that T^2 is a biased estimator for θ^2.

7. Explain most powerful test and uniformly most powerful test.

8. Let $X_1, X_2, \ldots X_n$ be a random sample from Cauchy population:

$$f(x, \theta) = \frac{1}{\pi} \cdot \frac{1}{1 + (x - \theta)^2}; -\infty < x < \infty; -\infty < \theta < \infty$$

Examine if there exists a sufficient statistic for θ.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If A and B are independent events, then prove that

(a) A and $\overline{B}$

(b) $\overline{A}$ and B

(c) $\overline{A}$ and $\overline{B}$ are also independent.

10. From a city population, the probability of selecting

   (a) a male or a smoker is 7/10,

   (b) a male smoker is 2/5, and

   (c) a male, if a smoker is already selected is 2/3.
   Find the probability of selecting

      (i) a non-smoker,

      (ii) a male, and

      (iii) a smoker, if a male is first selected.

11. State and Prove Chebychev's inequality.

12. State and Prove Lindeberg-Levy theorem.

13. If $X_1, X_2, ..., X_n$ are random observations on a Bernoulli variate X taking the value 1 with probability P and the value 0 with probability (1- p), show that:

   $\frac{\sum x_i}{n}\left(1-\frac{\sum x_i}{n}\right)$ is a consistent estimator of $p(1-p)$

14. State and Prove Neymann Pearson Lemma.

15. Given the frequency function

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, 0 \le x \le \theta \\ 0, \text{elsewhere} \end{cases}$$

and that you are testing the null hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2,$, by means of a single observed value of x. What would be the sizes of the type I and type II errors, if you choose the interval (a) $0.5 \le x,$, (b) $1 \le x \le 1.5$ as the critical regions? Also obtain the power function of the test.

16. State and Prove Rao Cramer inequality.
