PG–708	MMS–25

M.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Second Year

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. Define:
   (a) Topology on a set X
   (b) Standard topology on the real line

2. Prove that every finite point set in a Hausdorff space X is closed.

3. Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

4. State and prove intermediate value theorem.

5. Prove that every metrizable space is normal.

6. State and prove Holder's inequality.

7. If P is a projection on a Branch space B and if M and N are its range and null spaces, then prove that M and N are closed linear subspace of B such that B=M⊕N.

8. State and prove Bessel's inequality.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let $\{X_\alpha\}$ be an indexed family of spaces. Let $A_\alpha C X_\alpha$ for each α. If π_α is given either the product or the box topology, then prove that $\pi\overline{A}_\alpha = \overline{\pi A_\alpha}$

10. If L is a linear continuum is the order topology, then prove that L is connected and so intervals and rays is L.

11. Let X be a simply ordered set having the least upper bound property. In the order topology, prove that each closed interval in X is compact.

12. Prove that every well-ordered set X is normal in the order topology.

13. State and prove Urysohn's metrization theorem.

14. State and prove open mapping theorem.

15. State and prove uniform boundedness theorem.

16. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

PG–709	**MMS–26**

M.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

1. Write the advantages of Linear programming.

2. Write canonical and standard forms of LPP.

3. Write Minimal Spanning Tree Algorithm.

4. (a) State principle of optimality

   (b) Explain forward and backward Computational procedure.

5. Explain Branch and Bound method.

6. Solve the following game $\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$

7. State and prove forget fulness property.

8. Explain Direct search method.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Use simplex method to solve the following L.P.P.

$$\begin{aligned} &Max\ \ Z = 4x_1 + 10x_2 \\ &\text{Subject to } 2x_1 + x_2 \le 50 \\ &\qquad 2x_1 + 5x_2 \le 100 \\ &\qquad 2x_1 + 3x_2 \le 90 \\ &\qquad x_1, x_2 \ge 0 \end{aligned}$$

10. Consider the L.P.P

$$\begin{aligned} &Max\quad Z = 5x_1 + 12x_2 + 4x_3 \\ &\text{Subject to } \ x_1 + 2x_2 \le 5 \\ &\qquad 5x_1 - x_2 + 2x_3 = 2 \\ &\qquad x_1, x_2, x_3 \ge 0 \end{aligned}$$

Solve the above L.P.P.

11. Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project

Activity	1-2	1-3	1-5	2-3	2-4
Duration (in weeks)	8	7	12	4	10

Activity	3-4	3-5	3-6	4-6	5-6
Duration (in weeks)	3	5	10	7	4

12. Use dynamic programming to solve.

$Max\ Z = \ y_1\, y_2\, y_3$

Subject to $y_1 + y_2 + y_3 = 5$

$y; y_{2,} y_3 \geq 0$

13. Solve the following 2×4 game graphically

Player B

$$\text{Player A} \begin{pmatrix} 1 & 0 & 4 & -1 \\ -1 & 1 & -2 & 5 \end{pmatrix}$$

14. Use Branch and Bound technique to solve the following.

$Max\ Z = x_1 + 4x_2$

Subject to $2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

$x_1, x_2 \geq 0$ and are integers

15. Obtain the steady state differential difference equation for the model $(m/m/1):(\infty/FCFS)$ and hence solve its.

16. Solve the following problem by geometric programming.

$$\text{Minimize } Z = 5x_1x_2^{-1} + 2x_1^{-1}x_2 + 5x_1 + x_2^{-1}$$
$$x_1, x_2 \geq 0.$$

PG–710	**MMS–27**

M.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Second Year

Mathematics

GRAPH THEORY AND ALGORITHMS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that in any graph, the number of vertices of odd degree is even.

2. Write Dijkstra's algorithm.

3. Prove that if any two graphs G_1 and G_2 are isomorphic, then they are also 1-isomorphoic.

4. Define
   (a) Euler tour
   (b) Hamiltonian cycle

5. (a) Define maximal matching and give an example.
   (b) State the marriage problem

6. Prove that a graph G is bipartite if and only if G is bichromatic.

7. If G is a simple planar graph, then prove that G has a vertex of degree less than 6.

8. Prove that K_5 is non-planar.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Write down Kruskal's algorithm.

10. State and prove Cayley's formula.

11. Prove that a graph of G is bipartite if and only if G contains no odd cycles.

12. State and prove Vizing's theorem.

13. State and prove Brook's theorem.

14. State and prove four colour theorem.

15. Prove that $K_{3,3}$ is non-planar.

16. Write down sequential colouring algorithm.

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PG – 711	**MMS–28**

M.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Second Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions

1. Solve $x'''+6x''11x'+6x = 0, -\infty < t < \infty$.

2. Find the Wronksian of the functions $e^t, \cos t, \sin t$.

3. Use the method of variation of parameters to find a general solution of $x''+x = \tan t$.

4. Find the power series solution of the Legendre equation $(1-t^2)x''-2tx'+p(p+1)x = 0$ where p is a real number.

5. If $P_n(t)$ and $P_m(t)$ are Legendre polynomial, prove that $\int_{-1}^{1} P_n(t) P_m(t) dt = 0, m \neq n$.

6. Let $f(t)$ be periodic with period ω. Prove that a solution $x(t)$ of $x' = Ax + f(t), t \in (-\infty, \infty)$ is periodic of period ω if and only if $x(0) = x(\omega)$.

7. Find a solution of $(D^2 - D')z = 2y - x^2$.

8. Solve $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2 \frac{\partial^4 z}{\partial x^2 \partial y^2}$.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove Abel's formula for Wronskian.

10. State and prove Existence and uniqueness theorem of initial value problem.

11. If $a_1, a_2, a_3, \ldots$ be the positive zeros of the Bessel function $J_p(t)$ show that

$$\int_0^1 t Jp(a_m t) J_p(a_n t) dt = \begin{cases} 0 & m \neq n \\ \frac{1}{2} J_p^2(a_n), & m = n. \end{cases}$$

12. Determine the fundamental matrix for the system

$$x' = Ax, \text{ where } A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}.$$

13. State and draw Picard's theorem.

14. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

15. Derive Bessel's function of order 0 first kind.

16. State and prove Kelvin's inversion theorem.
