PG–377 PGDMAT–11/ MMS–15

P.G. DIPLOMA IN MATHEMATICS EXAMINATION – JUNE 2019.

ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that a non-empty subset H of the group G is a subgroup of G if and only if

   (a) $a, b \in H$ implies that $\dot{a}\dot{b} \in H$.

   (b) $a \in H$ implies that $a^{-1} \in H$.

2. If G is a finite group and $a \in G$, then prove that $O(a)/O(G)$.

3. If R is a ring, then prove that for all $a, b \in R$

   (a) $AO = OA = 0$.

   (b) $a(-b) = (-a)b = -(ab)$.

4. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if $\frac{R}{M}$ is a field.

5. If V is the internal direct sum of $U_1, U_2,..,U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2,..,U_n$ where $U_1, U_2,..,U_n$ are subspaces of the vector space V.

6. If V is a finite dimensional inner product space and W is a subspace of V, then prove that $W^{\perp})^{\perp} = W$.

7. If $f(x) \in F[x]$ is of degree $n \geq 1$, then prove that there is an extension E of F of degree atmost n ! in which $f(x)$ has n-roots.

8. If $T \in A(V)$ and $S \in A(V)$ is regular, then prove that STS^{-1} and T have the same minimal polynomial

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove Lagrange's theorem.

10. If G is a finite group, then prove that $C_a = O(G)/O(N(a))$.

11. Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring *R*. If and only if a_o is a prime element of *R*.

12. If *V* is a finite dimensional vector space and if *W* is a subspace of *V*, then prove that *W* is finite dimensional, $\dim W \leq \dim V$ and $\dim V / W = \dim V - \dim W$.

13. If *V* and *W* are of dimensions *m* and *n* respectively over *F*, then prove that $Hom(V, W)$ is of dimension *mn* over *F*.

14. If *F* is of characteristic '*O*' and if a, b are algebraic over *F*, then prove that there exists an element $C \in F(a, b)$ such that $F(a, b) = F(c)$.

15. (a) Prove that element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if there exists some $v \neq 0$ in *V*, $vT = \lambda v$.

    (b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then prove that λ is a root of the minimal polynomial of *T*.

16. Prove that there exists a subspace *W* of *V* invariant under *T* such that $V = V_1 \oplus W$.

PG–378	**MMS-16 / PGDMAT–12**

M.Sc. (Mathematics)/P.G. DIPLOMA IN MATHEMATICS EXAMINATION – JUNE, 2019.

First Year

REAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. State and prove the Archimedean property of R.

2. Prove that a set E is open if and only if its compliment is closed.

3. Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that $f(X)$ is compact.

4. Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f is discontinuous is atmost countable.

5. Let f be defined on $[a, b]$. If f has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists then prove that $f'(x) = 0$.

6. If p^* is a refinement of a partition p, then prove that $L(p, f, \alpha) \leq L(p^*, f, \alpha)$.

7. State and prove the fundamental theorem of calculus.

8. If $x > 0$ and $y > 0$, then prove that

$$\int_0^1 t^{x-1}(1-t)^{y-1}\, dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = p(x, y).$$

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. (a) Prove that closed subsets of compact sets are compact.

(b) Prove that the series $\sum a_n$

(i) Converges if $\lim\limits_{n\to\infty} \sup \left|\frac{a_{n+1}}{a_n}\right| < 1$,

(ii) Diverges if $\left|\frac{a_{n+1}}{a_n}\right| \geq 1$ for all $n \geq n_0$, where n_0 is some fixed integer.

10. Suppose $\{s_n\},\{t_n\}$ are complex sequences and $\lim_{n\to\infty} s_n = S$, $\lim_{n\to\infty} t_n = t$ Then prove that

(a) $\lim_{n\to\infty}(s_n + t_n) = s + t$

(b) $\lim_{n\to\infty} cs_n = cs$, $\lim_{n\to\infty}(c + s_n) = c + s$ for any number c.

(c) $\lim_{n\to\infty} s_n t_n = st$.

(d) $\lim_{n\to\infty} \frac{1}{s_n} = \frac{1}{s}$ provided $s_n \neq 0$ $(n = 1, 2, 3, ...)$ and $s \neq 0$.

11. (a) Define a continuous map on a metric space X.

(b) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that if is uniformly continuous on X.

12. State and prove the Taylor's theorem.

13. Assume α increases monotonically and $\alpha' \in R$ on $[a, b]$. Let f be a bounded real function on $[a, b]$. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case $\int_a^b f\,dx = \int_a^b f(x)\alpha'(x)dx$.

14. State and prove the Stone-Weierstrass theorem.

15. Suppose $\sum c_n$ converges, put

$f(x) = \sum_{n=0}^{\infty} c_n x^n \ (-1 < x < 1)$ then prove that $\lim_{x \to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

16. (a) Define a contraction map.

(b) If X is a complete metric space and ϕ is a contraction of X into X, then prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

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PG–379	MMS–17

M.Sc. DEGREE EXAMINATION — JUNE, 2019.

First Year

Mathematics

COMPLEX ANALYSIS AND NUMERICAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Find the bilinear transformation which maps the points $z = -z, 0, z$ into the points $w = 0$, $\bar{z}, -\bar{z}$ respectively.

2. State the prove fundamental theorem of algebra.

3. Obtain the Taylor's series for $\frac{z^2 - 1}{(z+2)(z+3)}$ is the region $|z| < 2$.

4. Find the roots of the equation $2x = \cos x + 3$ correct to three decimal places by iterative method.

5. Solve the following system of linear equations, using Gauss-Elimination method

$$x + 6y + 3z = 6$$
$$2x + 3y + 3z = 117$$
$$4x + y + 2z = 283$$

6. If $f(x) = \frac{1}{x^2}$ then find the divided differences $f(a, b)$ and $f(a, b, c)$.

7. Given that :

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ at $x = 1.6$.

8. Using Taylor's series find the solution of the differential equation $xy' = x - y$; $y(2) = 2$ at $x = 2.1$ correct the five decimal places.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Prove that the function $f(z) = e^x (x \cos y - y \sin y)$ satisfies Laplace's equation and find the corresponding analytic function.

10. Find all the bilinear transformations which transforms the unit circle $|z| \leq 1$ into the unit circle $|w| \leq 1$.

11. State and prove Rouche's theorem.

12. Solve :

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

By the factorization method.

13. Find the dominant eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, taking $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

14. Derive Lagrange's interpolation formula.

15. If third differences are constant, prove that

$$y_{x+\frac{1}{2}} = \frac{1}{2}(y_x + y_{x+1}) - \frac{1}{16}(\Delta^2 y_{x-1} + \Delta^2 y_x).$$

16. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.

PG–380	**MMS–18**

M.Sc. DEGREE EXAMINATION —
JUNE, 2019.

First Year

Mathematics

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

1. State and prove Multiplication theorem of probability.

2. For any two events A and B, prove that $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with continuous density. Show that $Y_1 = \min(X_1, X_2, \ldots, X_n)$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ.

4. Let $X_1, X_2, \ldots$ be a *i.i.d.* Poisson variates with parameter λ. Use CLT to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + \ldots + X_n$; $\lambda = 2$ and $n = 75$.

5. Briefly explain about the characteristics of Estimators.

6. Write the 95% and 99% confidence interval for single mean.

7. Explain two types of errors in statistical hypothesis.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from Bernoulli distribution

$$f(x, \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}; & x = 0,1 \\ 0, & \text{otherwise} \end{cases}$$

Show that $\sum_{i=1}^{n} X_i$ is a complete sufficient statistic for θ.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. The diameter say X of an electric cable, is assumed to be a continuous random variable with p.d.f. : $f(x) = 6x(1-x), 0 \le x \le 1$

(a) Check that the above if p.d.f

(b) Obtain an expression for the c.d.f. of X

(c) Compute $P\left(X \le \frac{1}{2} \middle| \frac{1}{3} \le X \le \frac{2}{3}\right)$ and

(d) Determine the number k such that $P(X < k) = P(X > k)$.

10. State and prove Chebychev's inequality.

11. Write the cumulative distribution function of a single order statistic.

12. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with p.d.f. $f(x, \theta) = e^{-(x-\theta)}$, $\theta < x < \infty$, $-\infty < \theta < \infty$. Find a sufficient statistic for θ.

13. State and prove Neymann Pearson Lemma.

14. Prove that every Most Powerful (MP) or Uniformly Most Powerful (UMP) critical region (CR) is necessarily unbiased

(a) If W be an MPCR of a size α for testing $H_0 : \theta = \theta_1$, then it is necessarily unbiased

(b) If W be an UMPCR of a size α for testing $H_0 : \theta = \theta_1$ against $H_1 : \theta \in \theta_1$, then it is also unbiased.

15. State and prove Rao Cramer inequality.

16. Obtain the MVB estimator for μ in normal population $N(\mu, \sigma)$, where σ^2 is known.
