UG–339	BMS–35

B.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Third Year

Mathematics

GRAPH THEORY

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Show that in a graph G the number of vertices of odd degree is even.
2. Define Degree sequence of a graph.
3. Define Hamiltonian graph.
4. If G is k–critical, then prove that $\delta \geq k-1$.
5. Define Directed graph.
6. Define planar graph.
7. Show that the partition $P(6, 6, 5, 4, 3, 3, 1)$ is not graphic.
8. Prove that every Hamiltonian graph is 2 – connected.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. If G_1 be a (p_1, q_1) graph G_2 be a (p_2, q_2) graph then prove that $G_1 \times G_2$ is a $(p_1\ p_2,\ p_2 q_1 + p_1 q_2)$ graph.

10. Prove that if G is a tree with p vertices and q edges then the following statements are equivalent.

    (a) Every two vertices are connected by a unique path.

    (b) G is connected and $q = p - 1$

    (c) G is acyclic and $q = p - 1$

11. Prove that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

12. Prove that K , is nonplanar.

13. Prove that every tournament has a directed Hamilton path.

14. Prove that a graph G with p vertices $\delta \geq \frac{p-1}{2}$ is connected.

15. Prove that every uniquely $n-$colourable graph is $n-1$ connected.

16. Prove that any self complementary graphs has $4n$ or $4n+1$ points.

UG–338	BMS–34

B.Sc. DEGREE EXAMINATION — DECEMBER 2019.

Third Year

Mathematics

PROGRAMMING IN C AND C++

Time : 3 hours Maximum marks 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Explain the different types of operators in C with examples.

2. Discuss C storage class with example?

3. Explain a function with argument and return values with example.

4. Differentiate between call by value and call by reference with example.

5. Explain the switch statement and syntax and example.

6. What is friend function and inline function? Explain with example.

7. Write a C program to find greatest of three number using nested if.

8. Write a program to read a line of text and output the number of words and characters in it.

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE of the following.

9. What are the various data types available in C? Explain the memory size and range of data possible.

10. What is an array? Explain the declaration and initialization of one and two dimensional arrays with example.

11. Explain array of structures and structure within a structure with examples.

12. What are the types of files available in C? Explain the various file modes in C.

13. Explain different forms of inheritance. Illustrate each type with an example.

14. Explain the concept of pointers. Also explain operations on pointers and array of pointers.

15. Explain the different types of constructors with suitable program segments.

16. Write a C program to perform multiplication of two given matrices.

UG–336	**BMS–33/ BMC–33**

B.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Third Year

Mathematics/Mathematics with Computer Applications

LINEAR PROGRAMMING AND OPERATION RESEARCH

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Solve the following LPP by simplex method:

$$\text{Minimize } Z = 8x_1 - 2x_2$$
$$\text{Subject to } -4x_1 + 2x_2 \le 1$$
$$5x_1 - 4x_2 \le 3.$$
$$\text{and } x_1, x_2 \ge 0.$$

2. Using dual simplex method solve the following LPP:

$$\text{Minimize } Z = x_1 + x_2$$
$$\text{Subject to } 2x_1 + x_2 \ge 2$$
$$-x_1 - x_2 \ge 1$$
$$\text{and } x_1, x_2 \ge 0.$$

3. Write down the difference between the assignment problem and the transportation problem.

4. Find the initial basic feasible solution for the following transportation problem by Least Cost Method.

	To				Supply
From	1	2	1	4	30
	3	3	2	1	50
	4	2	5	9	20
Demand	20	40	30	10	

5. Solve the following 2 × 2 game :

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix} \end{array}$$

6. An item produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs.100 per set up and holding cost is Rs.0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

7. Derive the formula for average number of customers in the queue for multiple server and finite capacity.

8. Find the initial basic feasible solution for the following transportation problem by North West Corner method.

	To			Supply
From	1	2	6	7
	0	4	2	12
	3	1	5	11
Demand	10	10	10	

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Use Big – M method to solve

$$\text{Minimize } Z = 4x_1 - 3x_2$$
$$\text{Subject to } -2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6.$$
$$x_1 + x_2 \geq 6.$$
$$\text{and } x_1, x_2 \geq 0.$$

10. Use Gomory's cutting plane method to solve

$$\text{Maximize } Z = 2x_1 + 2x_2$$
$$\text{Subject to } 5x_1 + 3x_2 \leq 8$$
$$2x_1 + 4x_2 \leq 8.$$

and $x_1 > 0, x_2 \geq 0$ and are all integers.

11. Solve the following transportation problem by Vogel's approximation method.

		Destination				Supply
		A	B	C	D	
Source	1	11	20	7	8	50
	2	21	16	20	12	40
	3	8	12	18	9	70
Demand		30	25	35	40	

12. A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of the each job on each machine is given in the following table.

Machines					
		1	2	3	4
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

What are job assignments, which will minimize the cost? Find the minimum cost.

13. Reduce the following game by dominance and find the game value:

Player B

Player A		I	II	III	IV
	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

14. Find the optimal order quantity for a product for which the price – break as follows :

Quantity	Unit Cost
$0 \leq q_1 < 500$	Rs. 10
$500 \leq q_2 < 750$	Rs. 9.25
$750 \leq q_3$	Rs. 8.75

The monthly demand for the product is 200 units, the cost of the storage is 2% of the unit cost and ordering cost is Rs.100.00 per order.

15. A barber shop has 2 barbers and 3 chairs (totally 5 chairs) for waiting customers. Assume that customers arrive in Poisson fashion at a rate of 5 per hour and that each barber services customers according to exponential distribution with a mean of 1.5 minutes. Further, if a customer

arrives and there are no empty chairs in the shop he will leave. Find the steady state probabilities. What is the probability that the shop is empty? What is the expected number of customers in the shop?

16. Solve the following game by using simplex method.

$$\text{Player A} \overset{\text{Player B}}{\begin{bmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{bmatrix}}$$

UG–335 **BMS–32/BMC-32**

B.Sc. DEGREE EXAMINATION — DECEMBER, 2019.

Third Year

Mathematics

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let V be a vector space over a field F. Prove that

   (a) $\alpha 0 = 0$ for all $\alpha \in F$.

   (b) $0v = 0$ for all $v \in V$.

   (c) $(-\alpha)v = \alpha(-v) = -(\alpha v)$ for all $\alpha \in F$ and $v \in V$.

   (d) $\alpha v = 0 \Rightarrow \alpha = 0$ or $v = 0$.

2. Let A and B be subspace of a vector space V. Prove that $A \cap B = \{0\}$ if and only if every vector $v \in A + B$ can be uniquely expressed in the form $v = a + b$ where $a \in A$ and $b \in B$.

3. Find the linear transformation $T : V_3(R) \to V_3(R)$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard basis $\{e_1, e_2, e_3\}$.

4. Let $T : V \to W$ be a linear transformation. Prove that $\dim V = rank\, T + nullity\, T$.

5. Let V be a vector space of polynomials with inner product given by $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. If $f(t) = t + 2$, and $g(t) = t^2 - 2t - 3$, find $\langle f, g \rangle$.

6. Let V be a finite dimensional inner product space and let W be a subspace of V. Prove that $(W^{\perp})^{\perp} = W$.

7. Let f be a symmetric bilinear form defined on V and let q be the associated quadratic form. Prove that $f(u, v) = \frac{1}{4}[q(u+v) - q(u-v)]$.

8. Let G be a group and let L be the set of all subgroups of G. In L we define $A \leq B$ if and only if $A \subseteq B$. Prove that L is a lattice.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let V and W be vector spaces over a field F. Let $L(V,W)$ represent the set of all linear transformation from V to W. Prove that $L(V,W)$ itself is a vector space over addition and scalar multiplication defined by $(f+g)(v) = f(v) + g(v)$ and $(\alpha f)(v) = \alpha f(v)$.

10. Let V be a vector space over a field F. Let $S,T \subseteq V$. Prove that

    (a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

    (b) $L(SUT) = L(S) + L(T)$.

11. If W is a subspace of a finite dimensional vector space V, show that $\dim \frac{V}{W} = \dim V - \dim W$.

12. Let W_1 and W_2 be subspaces of a finite dimensional inner product space. Prove that

    (a) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$.

    (b) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$.

13. Apply Gram – Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$ where $v_1 = \{1, -1, 0\}$, $v_2 = \{2, -1, -2\}$ and $v_3 = \{1, -1, -2\}$.

14. Let V be a vector space over a field F. Prove that $L(V,V,F)$ is a vector space over F under addition and scalar multiplication defined by

    (a) $(f+g)(u,v) = f(u,v) + g(u,v)$ and

    (b) $(\alpha f)(u,v) = \alpha f(u,v)$, where $L(V,V,F)$ is the set of all bilinear form on V.

15. Reduce the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$ to the diagonal form.

16. (a) Let B be a Boolean algebra. Show that $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$ and $(a')' = a$.

    (b) In a Boolean algebra if $a \vee x = b \vee x$ and $a \vee x = b \vee x$ then show that $a = b$.

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B.Sc. DEGREE EXAMINATION –
DECEMBER 2019.

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the countable union of countable sets is countable.

2. Let f be a continuous function from a metric space M_1 into a metric space M_2. If M_1 is connected, then prove that the range of f is also connected.

3. Let A be a subset of the metric space $\langle M,\rho\rangle$. If $\langle A,\rho\rangle$ is compact, then prove that A is a closed subset of $\langle M,\rho\rangle$.

4. Find the Taylor's series about $x=2$ for $f(x)=x^3+2x+1,(-\infty<x<\infty)$.

5. Prove that the Cauchy - Riemann equations in polar coordinates are $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r}=-\frac{1}{r}\frac{\partial u}{\partial \theta}$.

6. Define invariant points of the transformation $w=\frac{az+b}{cz+d}$. Find the invariant points of the transformation $w=-\frac{2z+4i}{iz+1}$.

7. State and prove Liouville's theorem.

8. Find the Laurent's series of $f(z)=\frac{1}{z(1-z)}$ valid in $|z+1|<1$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. State and prove Holder's inequality.

10. Prove that the metric space $\langle M,\rho\rangle$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

11. State and prove Lagrange's mean value theorem.

12. State and prove second fundamental theorem of integral calculus.

13. Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Also find the conjugate harmonic function v and the corresponding analytic function *f(z)*.

14. Find the bilinear transformation which maps the points 0, –i, –1 of the z – plane into the points i, 1, 0 of the w – plane respectively.

15. State and prove Cauchy's integral formula.

16. Evaluate $\int_0^{\infty} \frac{x \sin x}{(x^2 + a^2)} dx$, using contour integration.
