PG-768	PGDMAT-13

P.G. DIPLOMA EXAMINATION – DECEMBER 2019.

Mathematics

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that every finite point set in a Hausdorff space X is closed.

2. State the prove the pasting lemma.

3. Show that compactness implies the limit point compactness but not conversely.

4. If X has a countable basis, then prove that there exists a countable subset of X that is dense in X.

5. Prove that the linear space of all n-tuples $x = (x_1 x_2 .....x_n)$ of scalars denoted by l_∞^n whose norm defined by $\|x\| = \max\{|x_1|, |x_2|,...,|x_n|\}$ is a Banach space.

6. State

   (a) The Hahn-Banach theorem

   (b) The open mapping theorem.

7. State and prove the Parallelogram law in a Hilbert space H.

8. If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. (a) If Y is a subspace of X, then prove that the subset A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

   (b) State and prove the sequence lemma. (5+5)

10. Prove that the product of finitely many compact spaces is compute.

11. List X be a non-empty Hausdorff space. If X has no isolated points, then prove that X is countable.

12. Prove that a subspace of a Hausdorff space is Hausdorff and a product of Hausdorff spaces is Hausdorff.

13. If B and B' are Banach spaces and if T is a continuous linear transformation of B onto B', then prove that the image of each open sphere centred on the origin in B contains an open sphere centred on the origin in B'.

14. (a) State the closed graph theorem.

    (b) State and prove the uniform Boundedness theorem. (2+8)

15. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

16. Let H be a Hilbert space and let f be an arbitrary functional in H^*. Then prove that there exists a unique vector y in H such that $f(x) = \langle x, y \rangle$ for every x in H.