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| **UG–331** | **BMC–21/ BMS–21** |

# B.Sc. DEGREE EXAMINATION – JUNE, 2019.

Second Year

Mathematics with Computer Applications

## GROUPS AND RINGS

Time : 3 hours — Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Define (a) partial ordering relation, (b) poset and give examples.

2. Define centre of a group and normalizer of a group.

3. If $H$ and $K$ are subgroups of a group $G$, then show that $H \cap K$ is also a subgroup of $G$.

4. Let $H$ be a subgroup of a group $G$. Show that the number of left cosets of $H$ is the same as the number of right cosets of $H$.

5. Define a normal subgroup of a group. Show that every subgroup of an abelian group is a normal subgroup.

6. Let $R$ be a ring with identity. Show that the set of all units in $R$ is a group under multiplication.

7. Show that a finite commutative ring R without zero divisor is a field.

8. Show that the field of complex numbers is not an ordered field.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Define a bijection. Let $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x-3$. Show that $f$ is a bijection and compute $f^{-1}$. Also compute $f^{-1}\circ f$ and $f\circ f^{-1}$.

10. Show that a non-empty subset $H$ of a group $G$ is a subgroup of $G$ if and only if $a, b\in H \Rightarrow ab^{-1}\in H$.

11. State and prove Lagrange's theorem.

12. State and prove Cayley' s theorem.

13. Let $R$ be a commutative ring with identity. Show that an ideal $M$ of $R$ is maximal if and only if $R/M$ is a field.

14. (a) Show that any field is an integral domain.

    (b) Show that any finite integral domain is a field.

15. Show that any integral domain $D$ can be embedded in a field $F$ and every element of $F$ can be expressed as a quotient of two elements of $D$.

16. Show that any Euclidean domain is a unique factorization domain.

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# B.Sc. DEGREE EXAMINATION – JUNE, 2019.

Second Year

Mathematics

## STATISTICS AND MECHANICS

Time : 3 hours Maximum marks : 75

### SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. From the following data compute arithmetic mean :

| Marks : | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 |
|---|---|---|---|---|---|---|
| No. of students : | 5 | 10 | 25 | 30 | 20 | 10 |

2. Fit a straight line to the following data :

| Year : | 1969 | 1970 | 1971 | 1972 | 1973 | 1974 | 1975 | 1976 |
|---|---|---|---|---|---|---|---|---|
| Sales (in lakhs of Rs. ): | 38 | 40 | 65 | 72 | 69 | 60 | 87 | 95 |

Estimate the sales for 1977.

3. Calculate Spearman's coefficient of correlation between marks assigned to ten students by judges X and Y in a certain competitive test as shown below.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks by judge X : | 52 | 53 | 42 | 60 | 45 | 41 | 37 | 38 | 25 | 27 |
| Marks by judge Y : | 65 | 68 | 43 | 38 | 77 | 48 | 35 | 30 | 25 | 50 |

4. Compute Fisher's Ideal Index number from the following data :

| Commodity | 1999 | | 2000 | |
|---|---|---|---|---|
| | Price | Quantity | Price | Quantity |
| A | 2 | 8 | 4 | 6 |
| B | 5 | 10 | 6 | 5 |
| C | 4 | 14 | 5 | 10 |
| D | 2 | 19 | 2 | 13 |

5. A random variable $X$ has the following probability function values of $X$,

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ : | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $p(x)$ : | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2 + k$ |

(a) Find $k$,

(b) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$

6. Twenty people were attached by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease, is 85% in favour of the hypothesis that it is more, at 5% level (use large sample test)

7. A particle is projected over a triangle from one end of its horizontal base to graze the vertex and fall at the other end of the base. If $B$ and $C$ are the base angles and $\alpha$, the angle of projection show that $\tan\alpha = \tan B + \tan C$.

8. Show that the resultant of two simple harmonic motions is also simple harmonic.

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Calculate $\beta_1$ and $\beta_2$ from the following distribution :

| Age : | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 |
|---|---|---|---|---|---|---|---|---|
| Frequency : | 2 | 8 | 18 | 27 | 25 | 16 | 7 | 2 |

10. From the following data obtain the two regression equations and estimate the value of $Y$ which should correspond on an average to $X = 6.2$.

11. The following are the annual premiums charged by the Life Insurance Corporation of India for a policy of Rs. 1,000. Calculate the premium payable at the age of 26 by using Newton's formula.

| Age in Years : | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|
| Premium (Rs.) : | 23 | 26 | 30 | 35 | 42 |

12. The following table relates to the tourist arrivals during 1990 to 1996 in India :

| Years : | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 |
|---|---|---|---|---|---|---|---|
| Tourist arrivals (in millions) : | 18 | 20 | 23 | 25 | 24 | 28 | 30 |

Fit a straight line trend by the method of least squares and estimate the number of tourists that would arrive in the year 2000.

13. The incidence of occupational disease in an industry is that the workmen have a 20% chance of suffering from it. What is the probability that out of six workmen, 4 or more will contact the disease?

14. To assess the significance of possible variation in performance in a certain test between the grammar schools of a city a common test was given to a number of students taken at random from the senior fifth class of each of the four schools concerned. The results are given below :

Make an analysis of variance of data

Schools

| A | B | C | D |
|---|---|---|---|
| 8 | 12 | 18 | 13 |
| 10 | 11 | 12 | 9 |
| 12 | 9 | 16 | 12 |
| 8 | 14 | 6 | 16 |
| 7 | 4 | 8 | 15 |

15. Discuss the motion of two smooth spheres collide each other directly.

16. Obtain the differential equations of a central orbit in polar coordinates.

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# B.Sc DEGREE EXAMINATION – JUNE, 2019.

First Year

Mathematics/Mathematics with Computer Applications

## CLASSICAL ALGEBRA AND NUMERICAL METHODS

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Sum to infinity the series $1+\frac{1+3}{2\,!}+\frac{1+3+3^2}{3\,!}+\frac{1+3+3^2+3^3}{4\,!}+\dots$.

2. Prove that if $n>2$, then $(n\,!)^2>n^n$.

3. Solve the equation $x^3-12x^2+39x-28=0$ whose roots are in A.P.

4. Assuming that a root of $x^3 - 9x + 1 = 0$ lies between 2 and 3, find that root by bisection method.

5. Solve the following system of equations by Gauss elimination method

$$2x + 3y - z = 5,\ 4x + 4y - 3z = 3,\ 2x - 3y + 2z = 2$$

6. Use Lagrange's formula to fit a polynomial to the following data and hence find $y(1)$.

| $x$ | –1 | 0 | 2 | 3 |
|---|---|---|---|---|
| $y(x)$ | –8 | 3 | 1 | 12 |

7. Give the following table, find $f(35)$, by Stirling's formula of interpolation

| $x$ | 20 | 30 | 40 | 50 |
|---|---|---|---|---|
| $y(x)$ | 512 | 439 | 346 | 243 |

8. Find the first derivative of the function tabulated below at $x = 0.6$

| $x$ | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 |
|---|---|---|---|---|---|
| $y$ | 1.5836 | 1.7974 | 2.0442 | 2.3275 | 2.6511 |

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Find the sum to infinity the series $\frac{5}{3\cdot 6} + \frac{5\cdot 7}{3\cdot 6\cdot 9} + \frac{5\cdot 7\cdot 9}{3\cdot 6\cdot 9\cdot 12} + \dots$.

10. Sum to infinity the series $\frac{1}{1\cdot 2\cdot 3}+\frac{5}{3\cdot 4\cdot 5}+\frac{9}{5\cdot 6\cdot 7}+\ldots$.

11. Solve the equation $6x^5+11x^4-33x^3-33x^2+11x+6=0$.

12. Solve the following system of equations by using Gauss-Seidal method

    $8x-3y+2z=20$, $4x+11y-z=33$ and $6x+3y+12z=35$.

13. Using Newton-Raphson method, find the real positive root of $3x-\cos x-1=0$.

14. Interpolate by means of Gauss's backward formula the population of a place for the year 1966, given the following table :

| Year : | 1931 | 1941 | 1951 | 1961 | 1971 | 1981 |
|---|---|---|---|---|---|---|
| Population : (in thousands) | 12 | 15 | 20 | 27 | 39 | 52 |

15. Evaluate $\int_4^{5.2} \log_e x \, dx$ using :

    (a) Trapezoidal rule,

    (b) Simpson's one-third rule,

    (c) Simpson's three-eight rule.

16. Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given $y(0) = 1$ at $x = 0.2, 0.4$ .

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