UG–655	**BMS–31/ BMC–31**

B.Sc. DEGREE EXAMINATION — JUNE 2018.

Third Year

Mathematics

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Prove that the set $[0,1] = \{x : 0 \leq x \leq 1\}$ is uncountable.

2. If (M, ρ) is a complete metric space and A is a closed subset of M, then prove that (A, ρ) is also complete.

3. Let f be a continuous function from a metric space M_1 into a metric space M_2, If M_1 is connected then prove that the range of f is also connected.

4. Let f be a continuous real - valued function on the closed bounded interval $[a,b]$. If the maximum value of f is attained at c where $a < c < b$ and if $f'(c)$ exists then prove that $f'(c) = 0$.

5. Prove that $\sqrt{x}$ is a continuous function on $[0, \infty)$.

6. Find the value of a, b, c, d so that the function $f(z) = \left(x^2 + axy + by^2\right) + i\left(cx^2 + dxy + y^2\right)$ may be analytic.

7. Evaluate $\int_c \frac{z+1}{\left(z^3 - 2z^2\right)} dz$ where c is the circle $|z-2-i| = 2$ by using Cauchy's integral formula.

8. Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $1 + |z+1| < 2$.

PART B — (5 × 10 = 50 marks)

Answer any FIVE questions.

9. Let l^∞ denote the set of all bounded sequence of real numbers. If $x = \{x_n\}_{n=1}^\infty$ and $y = \{y_n\}_{n=1}^\infty$ are points in l^∞, define $\rho(x, y) = \underset{1 \le n < \infty}{l.u.b.} |x_n - y_n|$ then prove that $\left(l^\infty, \rho\right)$ is a metric space.

10. The metric space (M, ρ) is compact if and only if every sequence of points in M has a sequence converging to a point in M.

11. State and prove second fundamental theorem of calculus.

12. State and prove Taylor's formula with the integral form of the remainder.

13. State and prove a sufficient condition for differentiability of complex valued function.

14. Find the bilinear transformation which maps the points $0, -i$, -1 of the z-plane into the points i, 1, 0 of the w-plane respectively.

15. State and prove Rouche's theorem.

16. Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$, using contour integration, where $a > b > 0$.

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B.Sc. DEGREE EXAMINATION — JUNE 2018.

Third Year

Mathematics

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Let R^+ be the set of all positive real numbers. Define addition and scalar multiplication as follows:

   (a) $u + v = uv$ for all $u, v \in R^+$

   (b) $au = u^{\alpha}$ for all $u \in R^+$ and $\alpha \in R$.

   Prove that R^+ is a real vector space.

2. Show that the mapping $T : V_2(R) \to V_3(R)$ defined by $T(a, b) = (a + b, a - b, b)$ is a linear transformation.

3. Let V be a vector space over a field F. Show that any subsets of V containing the zero vector is linearly independent.

4. Show that every linearly independent subset of a finite dimensional vector space V forms a part of a basis.

5. Let $S = \{v_1, v_2, ...v_n\}$ be an orthogonal set of non — zero vectors in an inner product space V. Show that S is linearly independent.

6. Let f be the bilinear form defined on $V_2(R)$ by $f(x, y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f with respect to the standard basis $\{e_1, e_2\}$.

7. Define partial order relation on a set and give an example. Further check whether the relation 'a divides b' is a partial order on the set Z of integers.

8. Prove that any distributive lattice L is a modular lattice.

PART B — ($5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. State and prove fundamental theorem of vector space homomorphism.

10. Let V be a vector space over a field F. Let $S, T \subseteq V$. Prove that

(a) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$

(b) $L(SUT) = L(S) + L(T)$

11. Let V and W be vector spaces over a field F. Let $T : V \to W$ be an isomorphism. Prove that T maps a basis of V onto a basis of W.

12. Let V and W be two finite dimensional vector space over a field F. Let $\dim V = m$ and $\dim W = n$. Show that $L(V, W)$ is a vector space of dimension mn over F.

13. Apply Gram — Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis $\{v_1, v_2, v_3\}$. where $v_1 = \{1, 0, 1\}$. $v_2 = \{1, 3, 1\}$ and $v_3 = \{3, 2, 1\}$.

14. Show that the set of all bilinear forms on a vector space V is also a vector space over F.

15. Reduce the quadratic form $x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2$ to the diagonal form.

16. (a) Let B be a Boolean algebra. Show that $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$ and $(a')' = a$.

(b) In a Boolean algebra if a $a \vee x = b \vee x$ and $a \vee x' = b \vee x'$ Show that $a = b$.

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B.Sc. DEGREE EXAMINATION —
JUNE, 2018.

Third Year

Mathematics With Computer Application

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time : 3 hours Maximum marks : 75

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE questions.

1. Write basic assumptions in Linear Programming Models.

2. Write the dual of problem

$$Max\ Z = 2x_1 + 3x_2 + x_3$$

subject to $4x_1 + 3x_2 + x_3 = 6$

$$x_1 + 2x_2 + 5x_3 = 4$$

$$x_1, x_2, x_3 \geq 0$$

3. Define assignment problem and mention the necessary basic steps to solve it.

4. Find an initial basic feasible solution of the following transportation problem.

	D1	D2	D3	D4	
O1	1	2	1	4	30
O2	3	3	2	1	50
O3	4	2	5	9	20
	20	40	30	10	

5. Solve the game whose pay off matrix is

		I	II	III	B
	I	–3	–2	6	
A	II	2	0	2	
	III	5	–2	4	

6. Mention some of the advantages and disadvantages of having inventory.

7. Explain

   (a) Shortage Cost

   (b) Carrying Cost.

8. Explain Queue discipline.

SECTION B — $(5 \times 10 = 50$ marks)

Answer any FIVE questions.

9. Solve the following Linear programming problem by Simplex method.

$$Max\ Z = 5x_1 + 3x_2$$

subject to $3x_1 + 5x_2 \le 15$

$$5x_1 + 2x_2 \le 10$$

$$x_1, x_2 \ge 0.$$

10. Explain the concept of duality.

11. Solve the following transportation problem.

Source	Destination A	B	C	D	Supply
1	6	8	8	5	30
2	5	11	9	7	40
3	8	9	7	13	50
Demand	35	28	35	25	

12. Solve the following assignment problem

	J1	J2	J3	J4
A	10	15	24	30
B	16	20	28	10
C	12	18	30	16
D	9	24	32	18

13. Solve the game by graphical method

$$\begin{array}{ccc} & & \begin{array}{cccc} \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} \end{array} \\ \mathrm{A} & \begin{array}{c} \mathrm{I} \\ \mathrm{II} \end{array} & \begin{pmatrix} 1 & 4 & -2 & 3 \\ 2 & 1 & 4 & 5 \end{pmatrix} \end{array}$$

14. Explain basic classification of characteristics of Inventory systems.

15. Explain the queuing model $(M/M/1):(\infty/FCFS)$.

16. Prove that of arrival occur at random in time, the number of arrivals occuring in a fixed time interval follows a poisson distribution.

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B.Sc. DEGREE EXAMINATION — JUNE/JULY 2018.

Third Year

Mathematics

OPTIMIZATION TECHNIQUES

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Show that dual of a dual is a primal.

2. Write the standard form of the following :

   Maximize $Z = 5x_1 + 3x_2$

   Subject to the constraints

   $x_1 + x_2 \leq -4$

   $5x_1 + 2x_2 \leq 10$

   $3x_1 + 8x_2 \leq 12$

   and $x_1, x_2 \geq 0$

3. What is an assignment problem? How do balance an unbalanced assignment problem?

4. Write the principal assumptions made while dealing with sequencing problems.

5. Solve the following game and determine the value of the game

$$\begin{array}{cc} & B \\ A = & \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} \end{array}$$

6. Define the term economic order quantity and the term lead time.

7. An item is produced at the rate of 50 per day. The demand occurs at the rate of 25 per day. If the setup cost is Rs. 100 per setup and holding cost is Rs. 0.01 per unit per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the minimum total cost for one run.

8. In a super market there are two salesman. People arrive in a Poisson fashion at the rate of 10 per hour. The service time for each customer has a mean of 4 minutes. Then (a) Find the probability of having to wait for service. (b) What is the expected percentage of idle time for each salesman?

SECTION B — (5 × 10 = 50 marks)

Answer any FIVE of the following.

9. Use simplex method to solve the following linear programming problem.

Minimize $Z = x_1 - 3x_2 - 2x_3$

Subject to the constraints $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

and $x_1, x_2, x_3 \geq 0$

10. Find the integer solution to the linear programming problem.

Maximize $Z = 2x_1 + 2x_2$

Subject to the constraints

$5x_1 + 3x_2 \leq 8$, $x_1 + 2x_2 \leq 4$, $x_1, x_2 \geq 0$ and are integers.

11. Solve the following assignment problem.

3	5	10	15	8
4	7	15	18	8
8	12	20	20	12
5	5	8	10	6
10	10	15	25	10

12. Solve the following transportation by finding the initial solution using least cost method.

Destination

Origin	D_1	D_2	D_3	D_4	Availability
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Requirement	20	40	30	10	

13. Solve the game whose pay-off matrix is given below by graphical method.

$$\begin{array}{c c} & \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{pmatrix} 4 & -2 & 3 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 1 & -2 & 0 \end{pmatrix} \end{array}$$

14. Use dominance to reduce the following game to 2×2 game, and hence find the optimum strategies and the value of the game.

$$\begin{array}{c c} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 3 & -2 & 4 \\ -1 & 4 & 2 \\ 2 & 2 & 6 \end{pmatrix} \end{array}$$

15. Explain $(M/M/1):(\infty/FIFO)$ queuing model and solve it under steady state conditions.

16. Discuss an inventory model with several production runs of unequal length where shortages are not allowed.

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B.Sc. DEGREE EXAMINATION — JUNE, 2018.

First Year

Mathematics

PROGRAMMING IN C AND C++

Time : 3 hours Maximum marks : 75

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Explain the *printf* function in C in details.

2. Write a short note on *break* and *continue* statements in C.

3. How do you declare and initialize one-dimensional arrays in C.

4. Write a short note on bitwise operators in C.

5. What is a pointer? How do you declare pointer variables?

6. What is the difference between structure and arrays? Explain the general for of a structure.

7. Write a short note on

   (a) fopen ( )

   (b) feof ( ).

8. Explain the concept of function overloading in C++.

SECTION B — ($5 \times 10 = 50$ marks)

Answer any FIVE of the following.

9. Explain the fundamental data types in C.

10. Explain *for, while* and *do.... while* loops in C with example.

11. Explain recursion and write a program using recursion technique to find factorial of a given positive integer.

12. Write a C program to arrange the given set of *n* numbers in ascending order.

13. Explain pointer arithmetic in detail.

14. A file named DATA contains a series of integer numbers. Write a program to read those numbers and write them all odd numbers in a file to be called ODD and all even numbers to a file to be called EVEN.

15. Explain different types of inheritance that are available in C++.

16. What is constructor? Explain the concept of constructors with an illustration.

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UG–660	BMS–35

B.Sc. DEGREE EXAMINATION — JUNE/JULY, 2018.

Third Year

Mathematics

GRAPH THEORY

Time : 3 hours Maximum marks : 75

PART A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Prove that any self complementary Graph has $4n$ or $4n+1$ points.

2. Show that the sum of the degrees of the points of a graph G is twice as the number of lines.

3. Show that a graph G is a tree if and only if every two vertices of G are connected by a unique path.

4. Show that every connected graph G contains a spanning tree.

5. State and prove Euler formula for planar graphs.

6. Define edge coloring and vertex coloring of a graph.

7. Show that $k_{3,3}$ is nonplanar.

8. Check whether the sequence 4, 4, 4, 2, 2, 2 is graphic or not.

PART B — $(5 \times 10 = 50$ marks)

Answer any FIVE of the following.

9. (a) Show that the number of odd degree vertices in a graph G is even.

   (b) Show that a connected (p,q)-graph contains a cycle iff $q \geq p$.

10. Show that every nontrivial graph contains at least two vertices which are not cut vertices.

11. Show that a (p,q)- graph G is a bipartite graph iff it contains no odd cycles.

12. Show that a nontrivial connected graph is Eulerian if and only if it has no vertex of odd degree.

13. For a (p,q)-graph G, show that the following statements are equivalent.

    (a) G is a tree.

    (b) G is connected and $q = p-1$

    (c) G is a cyclic and $q = p-1$.

14. Show that every tournament contains a directed Hamilton path.

15. If G is a bipartite graph with $q(G) \geq 1$, then show that $\chi_1(G) = \Delta(G)$.

16. Prove that edge chromatic number of a complete graph on n if n is odd $(n \neq 1)$; $n-1$ if n is even.
